Slip-condition Effect on MHD Convective Flow through Porous Medium in a Vertical Channel with Thermal Radiation and Heat Source

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Abstract: The aim of this paper is to study the effects of slip-condition on the magnetohydrodynamic (MHD) convective flow of a viscous, incompressible and electrically conducting fluid through a porous medium filled between two infinite plates of the vertical channel in the presence of heat radiation and heat source/sink. One of these plates is subjected to a slip-flow condition at $y^* = -\frac{d}{2}$ and the other to a no-slip condition at $y^* = +\frac{d}{2}$. The temperature of the plate at $y^* = +\frac{d}{2}$ with no-slip condition is assumed to be varying in space and time both. The temperature difference of the walls of the channel is assumed high enough to induce heat transfer due to radiation. A magnetic field of uniform strength is applied perpendicular to the planes of the channel walls. The magnetic Reynolds number is assumed very small so that the induced magnetic field is neglected. The fluid is acted upon by spanwise sinusoidal fluctuating pressure gradient in the vertically upward direction. It is also assumed that the conducting fluid is optically-thin gray gas, absorbing / emitting radiation and non-scattering. Exact analytical solutions of the non-linear partial differential equations governing the flow problem are obtained. The velocity field, the temperature field, the amplitude and the phase angle of the skin friction and the heat transfer coefficient are shown graphically and their dependence on the various flow parameters is discussed in detail.

Keywords: slip-condition; convective flow; magnetohydrodynamic; spanwise sinusoidal; porous medium; heat radiation; heat source/sink.

Nomenclature

A	a constant	T_1, T_2	constant temperature
B_0	magnetic field applied	u, v, w	velocity comp. along X, Y, Z-axis
c_p	specific heat at constant pressure	V	injection/suction velocity
d	distance between plates	x, y, z	variables along X, Y, Z-directions
$\|F\|$	amplitude of skin-friction	α	mean radiation absorption coefficient

g	gravitational force	β	coefficient of volume expansion
Gr	Grashof number	σ^*	Stefan- Boltzmann constant
h	slip-flow parameter	λ	injection/suction parameter
$\lvert H \rvert$	amplitude of rate of heat transfer	ω	frequency of oscillations
K	permeability of porous medium	ϑ	kinematic viscosity
k	thermal conductivity	ρ	fluid density
L	mean free path	φ	phase angle of skin-friction
M	Hartmann number	ψ	phase angle of heat transfer
N	heat radiation parameter	τ_L	skin-friction at the left wall
p	pressure	θ_0	mean non-dimensional temperature
Pr	Prandtl number	Re	Reynolds number
Q^*	volumetric rate of heat generation.	t	time variable
T	fluid temperature	*	superscript for dimensional quantities

Introduction

The wall slip flow is a very important phenomenon that is widely encountered in this era of industrialization. It has numerous applications, for example in lubrication of mechanical devices where a thin film of lubricant is attached to the surface slipping over one another or when the surfaces are coated with special coatings to minimize the friction between them. By lubricating or coating the solid surface the fluid particles adjacent to it no longer move with the velocity of the surface but has a finite tangential velocity and, hence slips along the surface. A large number of scholars have shown their interest in the phenomenon of slip-flow regime due to its wide ranging applications.

Ahmed et al. [1] study the problem of MHD mixed convection and mass transfer flow of an electrically conducting incompressible viscous fluid past an infinite vertical porous plate with constant heat flux and chemical reaction in the presence of a heat source. Garg [3] discussed the combined effect of hall current and thermal radiations on unsteady flow of an electrically conducting flow past an impulsively started infinite vertical porous plate with variable temperature. Recently Garg also [4] analyzed MHD flow due to moving vertical porous plate with variable temperature in the presence of thermal radiations

Hamza et al. [5] investigated the effects of slip-condition, transverse magnetic field and radiative heat transfer on unsteady flow of an electrically conducting optically thin fluid through a porous medium filled in a channel. A three dimensional free convective and mass transfer slip flow in a porous medium with heat source is also analyzed by Jain and Vijay [7]. The temperature of the channel wall in these above listed problems is considered to be oscillating in time. In general there are majority of industrial situations e.g., boilers, furnaces, heat exchangers etc. where the temperature at different locations is different which further changes with time. In view of this a few studies have been conducted by considering spanwise cosinusoidal temperatures of the surfaces. Further by applying the perturbation technique Kumar et al. [9] investigated the same problem of slip-flow regime for the unsteady MHD periodic flow of viscous fluid through a planer channel. Kumar and Singh [10] studied an unsteady MHD flow of radiating and reacting fluid past a vertical porous plate with cosinusoidally fluctuating temperature.

There are many material processing industrial operations where due to high temperatures all the three modes of heat transportation such as conduction, convection and radiation accompany together. The important areas in which thermal radiation heat transfer must be considered along with thermal convection heat transfer are direct flame impingement (DFI) furnace for rapid heating of metals in materials processing (Malikov et al. [12]), heating of a continuously moving load in the industrial radiant oven (Fedorov et al. [2]), and glass melting simulation (Lentes and Siedow [11]).Taking heat radiation into account Mehmood and Ali [14] extended the problem of oscillatory MHD flow in a channel filled with porous medium studied by Makinde and Mhone [13] to slip-flow regime. Morques et al. [15] have considered the effect of the fluid slippage at the plate for Couette flow. Raptis et al. [16] studied the effects of radiation in an optically thin gray gas flowing past a vertical infinite plate in the presence of a magnetic field. Rhodes and Rouleau [17] studied the hydrodynamic lubrication of partial porous metal bearings.

The problem of the slip-flow regime plays a very important role in modern science, technology and vast ranging industrialization. In view of the practical applications of the slip-flow regime it remained of paramount interest for several scholars e.g. Sharma and Chaudhary [18]; Sharma [19]; Jain and Gupta [6]; Khaled and Vafai [8]; obtained exact solutions of oscillatory Stokes and Couette flows under slip flow condition. Singh [20] analyzed an unsteady free convection flow past a hot vertical porous plate with variable temperature. Singh and Khem Chand [21] discussed an unsteady free convective MHD flow past a vertical porous plate with such a variation of the temperature. Recently Singh [22] studied an oscillatory MHD forced convection flow of an electrically conducting, viscous incompressible fluid through a porous medium in a vertical channel under slip condition. He also [23] further obtained an exact solution of an oscillatory fully developed MHD convection flow through a porous medium in a vertical porous channel in slip flow regime. In these studies the temperature of one of the plates of the channel varies periodically with time only about a constant mean. Sumathi et al. [24] also attempted heat and mass transfer in an unsteady three dimensional mixed convection flow past an infinite vertical porous plate with cosinusoidally fluctuating temperature.

Hence, the aim of the present study is to formulate and analyze the very important physical problem of viscous, incompressible and finitely electrically conducting fluid flow through a porous medium bounded by two infinite vertical plates in the presence of heat radiation. The temperatures of channel plates with no-slip condition and with slip-condition respectively remain spanwise cosinusoidal and constant as shown in Fig.1a. A magnetic field of uniform strength is applied transverse to the flow and the magnetic Reynolds number is assumed very small so that the induced magnetic field is neglected. It is also assumed that the conducting fluid is optically-thin gray gas, absorbing/ emitting radiation and non-scattering. An exact solution of the mathematical problem so formed is obtained and the final results for the velocity, temperature, shear stress and heat transfer coefficient in terms of their amplitudes and phase angles are discussed in the last section of the paper.

Mathematical formulation

We consider an unsteady flow of a viscous, incompressible and electrically conducting fluid in a hot vertical channel filled with porous medium. The two parallel plates of the channel are distance 'd' apart. A Cartesian coordinate system (X^*, Y^*) is chosen such that X^*-axis directed upwards lies along the centerline of the channel and Y^*-axis is perpendicular to the planes of parallel plates. A magnetic field B_0 of uniform strength is applied transversely along Y^*-axis. The magnetic Reynolds number is assumed to be small enough so that the induced magnetic field is negligible. Hall Effect, electrical and polarization effects are also neglected. All the physical quantities except pressure are independent of x^* for this fully developed laminar flow in the infinite vertical channel. The temperature of the plate at $y^* = \frac{d}{2}$ varies as spanwise cosinusoidally, i.e. $T^* = T_1 + (T_2 - T_1)\cos\left(\frac{\pi z^*}{d} - \omega^* t^*\right)$. A schematic diagram of the physical problem with spanwise cosinusoidal variation of plate temperature is shown in Figs.1a. & 1b.

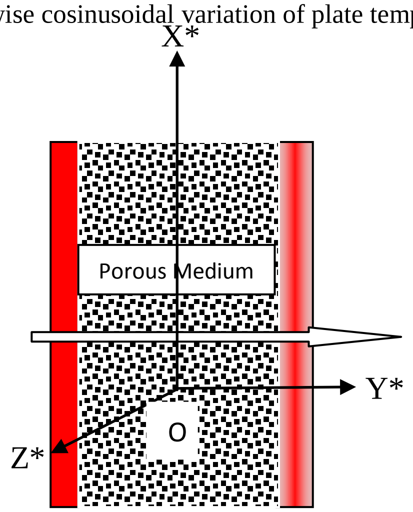


Fig.1a. Hot vertical channel

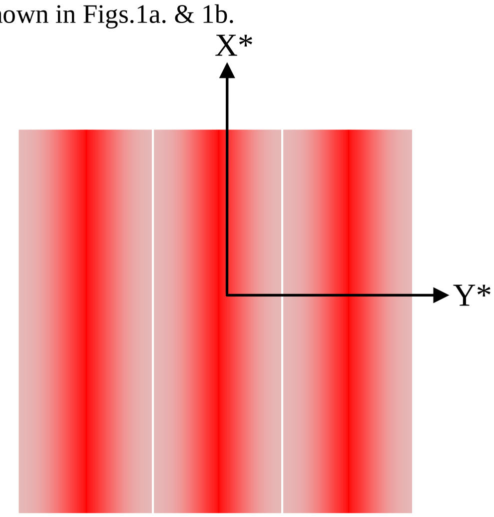


Fig.1b. Spanwise consinusiodal plate temperature

Then taking into account the usual Boussinsq's approximation, the forced and free convection flow is governed by the following differential equations:

Momentum equation;

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \vartheta\left(\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}\right) - \frac{\sigma B_0^2}{\rho}u^* - \frac{\vartheta}{K^*}u^* + g\beta(T^* - T_1), \tag{1}$$

Energy equation;

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right) - \frac{1}{\rho c_p}\frac{\partial q^*}{\partial y^*} - \frac{Q^*}{\rho c_p}(T^* - T_1)\ , \tag{2}$$

where in momentum equation (1) term on the L.H.S. is the inertial force and on the R.H.S. the terms respectively represent imposed pressure gradient, viscous force, Lorentz force due to magnetic field B_0, pressure drop across the porous matrix and the buoyancy force due to temperature difference. In energy equation (2) term on the L.H.S. is the heat due to convection and on the R.H.S. the terms respectively represent conduction heat, radiation heat and heat source/sink. All quantities have their usual meaning and Q^* is the volumetric rate of heat generation.

The boundary conditions of the problem are

$$u^* = L\frac{\partial u^*}{\partial y^*}, \quad T^* = T_1 \quad at \quad y^* = -\frac{d}{2}, \tag{3}$$

$$u^* = 0, \quad T^* = T_1 + (T_2 - T_1)\cos\left(\frac{\pi z^*}{d} - \omega^* t^*\right) \quad at \quad y^* = +\frac{d}{2}, \tag{4}$$

where $= \left(\frac{2-r_1}{r_1}\right)L_1$, L being the mean free path of the particle and r_1 the Maxwell's reflection coefficient.

For the case of an optically thin gray gas the local radiant is expressed by

$$\frac{\partial q^*}{\partial y^*} = 4a^*\sigma^*(T^{*4} - T_1^4), \tag{5}$$

where a^* is the mean absorption coefficient and σ^* is Stefan- Boltzmann constant. We assume that the temperature differences within the flow are sufficiently small such that T^{*4} may be expressed as a linear function of the temperature. This is accomplished by expanding T^{*4} in a Taylor series about T_1 and neglecting higher order terms, thus

$$T^{*4} \cong 4T_1^{*3}T^* - 3T_1^4. \tag{6}$$

Substituting (6) into (5) and simplifying, we obtain

$$\frac{\partial q^*}{\partial y^*} = 16a^*\sigma^*T_1^3(T^* - T_1). \tag{7}$$

Further, substitution of (7) into the energy equation (2) gives

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right) - \frac{16a^*\sigma^*T_1^3(T^*-T_1)}{\rho c_p} - \frac{Q^*}{\rho c_p}(T^* - T_1). \tag{8}$$

Now introducing the following non-dimensional quantities

$$x = \frac{x^*}{d},\ y = \frac{y^*}{d}, z = \frac{Z^*}{d},\ t = \frac{t^*U}{d},\ \omega = \frac{\omega^* d}{U},\ u = \frac{u^*}{U},\ \theta = \frac{T^*-T_1}{T_2-T_1},\ p = \frac{p^*}{\rho U^2}, \tag{9}$$

in equations (1) and (8) we obtain governing equations in dimensionless form as

$$Re\frac{\partial u}{\partial t} = -Re\frac{\partial p}{\partial x} + \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - (M^2 + K^{-1})u + Gr\,\theta, \tag{10}$$

$$RePr\frac{\partial \theta}{\partial t} = \left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}\right) - (N^2 + S)\,\theta, \tag{11}$$

with non-dimensional boundary conditions

$$u = h\frac{du}{dy}, \quad \theta = 0, \quad at \quad y = -\frac{1}{2}, \tag{12}$$

$$u = 0, \quad \theta = \cos(\pi z - \omega t), \quad at \quad y = \frac{1}{2}, \tag{13}$$

where $h = \frac{L}{d}$ is the slip-flow parameter,

$Re = \frac{Ud}{\vartheta}$ is the Reynolds number,

$Gr = \frac{g\beta(T_2 - T_1)d^2}{\vartheta U}$ is the Grashof number,

$M = B_0 d\sqrt{\frac{\sigma}{\mu}}$, is the Hartmann number,

$K = \frac{K^*}{d^2}$, is the permeability of the porous medium,

$Pr = \frac{\mu c_p}{k}$, is the Prandtl number,

$N^2 = \frac{16a^*\sigma^* T_1^3 d^2}{k}$, is the radiation parameter,

$S = \frac{Q^* d^2}{k}$ is the heat source/sink parameter

and U is the mean flow velocity.

Solution of the Problem

In order to obtain the solution of this unsteady problem it is convenient to adopt complex variable notations for velocity, temperature and pressure. The real part of the solution will have physical significance. Thus, we write velocity, temperature and pressure as

$$u(y,z,t) = u_0(y)e^{i(\pi z - \omega t)},\ \theta(y,z,t) = \theta_0(y)e^{i(\pi z - \omega t)},\ -\frac{\partial p}{\partial x} = Ae^{i(\pi z - \omega t)}, \tag{14}$$

where A is a constant.

The boundary conditions in equations (12) and (13) can also be written in complex notations as

$$u = h\frac{du}{dy}, \quad \theta = 0, \quad at \quad y = -\frac{1}{2}\ , \tag{15}$$

$$u = 0, \quad \theta = e^{i(\pi z - \omega t)}, \quad at \quad y = \frac{1}{2}\ . \tag{16}$$

Substituting expressions (14) into equations (10) and (11), we get

$$\frac{d^2u_0}{dy^2} - m^2u_0 = -ARe - Gr\theta_0, \tag{17}$$

$$\frac{d^2\theta_0}{dy^2} - n^2\theta_0 = 0\ , \tag{18}$$

where $m = \sqrt{(\pi^2 + M^2 + K^{-1} - i\omega Re)}$ and $n = \sqrt{(\pi^2 + N^2 + S - i\omega RePr)}$

with transformed boundary conditions as

$$u_0 = h\frac{du_0}{dy}, \qquad \theta_0 = 0 \quad at \quad y = -\tfrac{1}{2}, \tag{19}$$

$$u_0 = 0, \qquad \theta_0 = 1 \quad at \quad y = \tfrac{1}{2}. \tag{20}$$

The ordinary differential equations (17) and (18) are solved under boundary conditions (19) and (20) and the solutions for the velocity and the temperature fields are obtained, respectively, as

$$u(y,z,t) = \left[\begin{array}{c} \frac{ARe}{m^2}\left\{1 - \frac{2\,sinh\left(\frac{m}{2}\right) cosh(my) + mh\, cosh\left[m(y+\frac{1}{2})\right]}{(sinh(m)+mhcosh(m))}\right\} + \frac{Gr}{(n^2-m^2)\, sinh(n)} \\ \left\{\begin{array}{c} \frac{sinh(n)\left[sinh\left[m(y+\frac{1}{2})\right] + mh\, cosh\left[m(y+\frac{1}{2})\right]\right] + nhsinh\left[m(y-\frac{1}{2})\right]}{(sinh(m)+m\gamma cosh(m))} \\ -sinh\left[n(y+\frac{1}{2})\right] \end{array}\right\} \end{array}\right] e^{i(\pi z-\omega t)}, \tag{21}$$

$$\theta(y,z,t) = \left\{\frac{sinh\left[n(y+\frac{1}{2})\right]}{sinh(n)}\right\} e^{i(\pi z-\omega t)}. \tag{22}$$

From the velocity field in equation (21) we can obtain the skin-friction at the left wall, τ_L, in terms of its amplitude $|F|$ and the phase angle φ as

$$\tau_L = \left(\frac{\partial u_0}{\partial y}\right)_{y=-\frac{1}{2}} |F| \cos(\pi z - \omega t + \varphi)\ , \tag{23}$$

$$F_r + iF_i = \frac{ARe}{m^2}\left\{\frac{2msinh^2\left(\frac{m}{2}\right)}{sinh(m)+mhcosh(m)}\right\} + \frac{Gr}{(n^2-m^2)\, sinh(n)}\left\{\frac{m\left(sinh(n)+nhcosh(m)\right)}{sinh(m)+mhcosh(m)} - n\right\} \tag{24}$$

where the amplitude and the phase angle of the skin-friction τ_L are respectively given by

$$|F| = \sqrt{F_r^2 + F_i^2}\ , \quad \text{and} \quad \varphi = \tan^{-1}\left(\frac{F_i}{F_r}\right).$$

From the temperature field given in equation (22) the heat transfer coefficient Nu (Nusselt number) in terms of its amplitude and the phase angle can be obtained as

$$Nu = \left(\frac{\partial \theta}{\partial y}\right)_{y=-\frac{1}{2}} = |H| \cos(\pi z - \omega t + \psi), \tag{25}$$

where $H_r + i\, H_i = \frac{n}{\sinh n}$.

The amplitude $|H|$ and the phase angle ψ of the heat transfer coefficient Nu (Nusselt number) are given by

$$|H| = \sqrt{H_r^2 + H_i^2} \ \text{ and } \ \psi = \tan^{-1}\left(\frac{H_i}{H_r}\right) \ \text{ respectively.} \tag{26}$$

Results and Discussion

The problem of unsteady MHD radiative and convective flow through a porous medium in a vertical channel in the presence of heat source is analyzed. The closed form solutions for the velocity and temperature fields are obtained analytically and then evaluated numerically for different values of parameters appeared in the equations. To have better insight of the physical problem the variations of the velocity, temperature, skin-friction and the heat transfer coefficient in terms of their amplitudes and phase angles with the parameters involved in the flow problem are then shown graphically to assess the effect of each parameter.

Fig.2. shows the variation of the velocity with the slip-flow parameter h, Reynolds number Re, Grashof number Gr, Hartmann number M, permeability of the porous medium K, Prandtl number Pr, radiation parameter N, heat source S, pressure gradient A and the frequency of oscillations ω. The dotted curve I (….) represent the case of no-slip conditions (h = 0) on both the plates of the channel. The dashed

curve II (----) curve represent the flow with slip condition $(h \neq 0)$. To assess the influence of each of the parameter on the velocity every curve is compared with the dashed curve II (---). The comparison of curves III, IV, V, VII and XI with the dashed curve II (---) shows that the velocity increases with the increase slip-flow parameter h, the Reynolds number Re, Grashof number Gr, permeability of the porous medium K and favorable pressure gradient A respectively. It is expected physically also because a finite tangential velocity is introduced due to slip condition at the boundary. The increase of Reynolds number means the increase of inertial forces because of which velocity increases. The increase of velocity with the increase of the Grashof number Gr physically means that the enhancement of the buoyancy force leads to increase of the velocity component u(y, z, t) of the velocity. The increase of velocity with the increase of permeability of the porous medium indicates that the resistance posed by the porous medium reduces as the permeability of the medium increases because of which the velocity increases. As expected the larger favorable pressure gradient in the channel leads to faster flow, hence, velocity increases. The effects of other parameters like Hartmann number M, Prandtl number Pr, radiation parameter N, heat source S and frequency of oscillations ω are represented by curves VI, VIII, IX, X and XII respectively. Comparing these curves with the dashed curve II (---) shows that the flow velocity decreases with the increase of these parameters. Lorentz force which is introduced due to the application of the transverse magnetic field retards the velocity. This force gives a dragging effect on the flow. The two values of the Prandtl number chosen are Pr=0.7 and Pr=7 which represent air and water respectively, most commonly found fluids on the planet earth. It is evident that the velocity is less in water than in air. Since the Prandtl number gives the relative importance of viscous dissipation to the thermal dissipation so for larger Prandtl number viscous dissipation is predominant and due to this velocity decreases. The increase of radiation N, heat source S and the frequency ω lead to a decrease in velocity.

The skin-friction τ_L in terms of its amplitude $|F|$ and phase angle φ has been shown in Figs. 3. and 4. respectively. The sets of values of parameters listed in Table 2 are presented graphically in these figures. The effect of each of the parameter on $|F|$ and φ is assessed by comparing each curve with dashed curve I in these figures. In Fig.3. the comparison of curves III, IV, VI and IX with dashed curve I (---) indicates that the amplitude increases with the increase of Reynolds number Re, Grashof number Gr, the permeability of the porous medium K and the pressure gradient parameter A. It is expected physically also because due to the increase of these parameters the velocity increases and consequently the faster flows give rise to more skin-friction. Similarly the comparison of other curves II, V, VII, VIII and IX with the dashed curve I (---) depicts that the skin-friction amplitude decreases with the increase of slip parameter h, Hartmann number M, Prandtl number Pr, the radiation parameter N and heat source parameter S because the velocity due to these parameters decrease and for slow flows skin-friction is less except the case of slip parameter. The increase of velocity with increasing slip parameter is because of the tangential velocity at the plate which reduces the skin friction. It is obvious that amplitude $|F|$ remains almost constant with increasing frequency of oscillations ω except with the increase of Reynolds number Re Fig.4. shows the variations of the phase angle φ of the skin-friction with increasing frequency of oscillations ω. Here again the comparison of curves II, III, IV and VI with the dashed curve I (---) indicates that the phase increases with the increase of slip flow parameter h, Reynolds number Re, Grashof number Gr and the permeability of the porous medium K respectively. The comparison of curves namely V, VIII, IX and X with dashed curve I reveals that the phase angle decreases with the increase of Hartmann number M, the radiation parameter N, heat source S and pressure gradient A. From the behaviour of curve it is inferred that as the Prandtl number Pr increases the phase for small frequency ω increases but for large ω decreases. It is quite evident that there is always a phase lead because the values of φ remain positive throughout. It is also noticed from this figure that the phase lead goes on increasing with increasing frequency of oscillations ω.

Fig.5. illustrates the variation of the temperature field with the variation of different flow parameters. The comparison of different curves with the dashed curve I reveals that there is a decrease in temperature with the increase of any of the parameters like the Reynolds number Re, or the Prandtl number Pr or the radiation parameter N or the heat source S, or the frequency of oscillations ω.

The amplitude $|H|$ and the phase angle ψ of the Nusselt number (Nu), the heat transfer coefficient, against the frequency of oscillations ω are illustrated in Fig.6. and Fig.7. respectively for the sets of parametric values in Table 3. It is evident from Fig.6. that the amplitude $|H|$ decreases with the increase of Reynolds number Re, Prandtl number Pr, radiation parameter N and the heat source S. The amplitude in the case of water (Pr =7) decreases sharply with increasing frequency ω of oscillations and becomes negligible for larger values of ω. The amplitude $|H|$ becomes constant with increasing frequency of oscillations ω as the radiation parameter N and the heat source S increases. From Fig.7. it is obvious that there is always a phase lead because the values of ψ remain positive for all values of the frequency of oscillations ω. It is also noticed from this Figure that there is a linear increase in phase lead with the increasing frequency of oscillations ω as radiation and heat source increase. The phase angle ψ of the Nusselt number (Nu) starts oscillating between the phase lag and the phase lead as the Reynolds number or the Prandtl number are increased. For increasing Reynolds number the amplitude remains linear initially for smaller oscillations but oscillates thereafter for larger ω.

Table 1: Sets of parameter values shown graphically in Fig.2.

H	Re	Gr	M	K	P	N	S	A		Curve
0	1	1	2	0.2	0.7	1	0.5	2	1	I(…..)
0.2	1	1	2	0.2	0.7	1	0.5	2	1	II(----)
0.5	1	1	2	0.2	0.7	1	0.5	2	1	III
0.2	0.5	1	2	0.2	0.7	1	0.5	2	1	IV
0.2	1	5	2	0.2	0.7	1	0.5	2	1	V

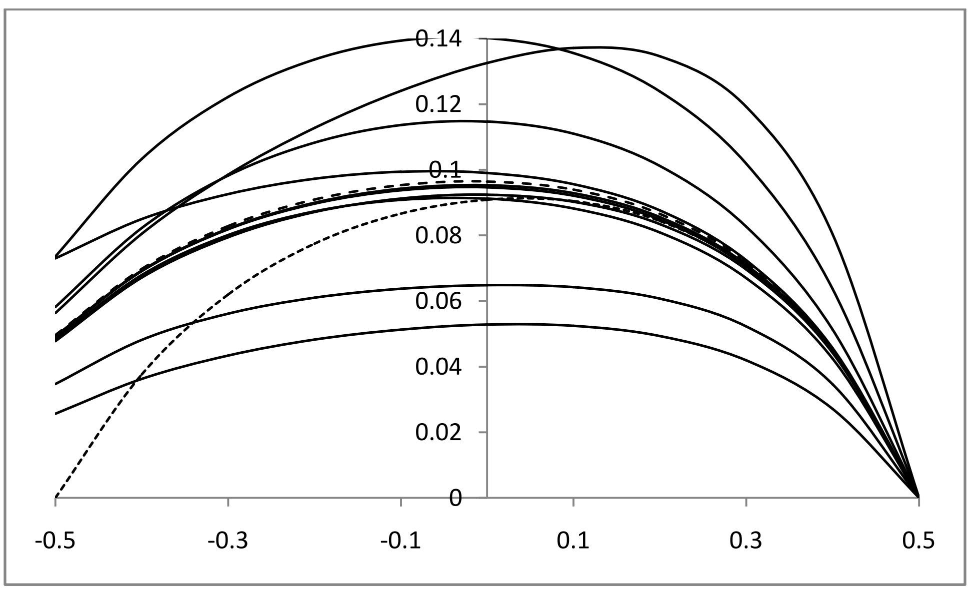


Fig.2. Velocity profiles for sets of parameter values in Table 1 for z=0.5 and t=π/2.

0.2	1	1	4	0.2	0.7	1	0.5	2	1	VI
0.2	1	1	2	1.0	0.7	1	0.5	2	1	VII
0.2	1	1	2	0.2	7.0	1	0.5	2	1	VIII
0.2	1	1	2	0.2	0.7	5	0.5	2	1	IX
0.2	1	1	2	0.2	0.7	1	5.0	2	1	X
0.2	1	1	2	0.2	0.7	1	0.5	3	1	XI
0.2	1	1	2	0.2	0.7	1	0.5	2	5	XII

Table 2. Sets of parameter values shown graphically in Figs.3. & 4.

H	Re	Gr	M	K	Pr	N	S	A	Curve
0.2	0.5	1	2	0.2	0.7	1	0.5	2	I(---)
0.5	0.5	1	2	0.2	0.7	1	0.5	2	II
0.2	1.0	1	2	0.2	0.7	1	0.5	2	III
0.2	0.5	5	2	0.2	0.7	1	0.5	2	IV
0.2	0.5	1	4	0.2	0.7	1	0.5	2	V
0.2	0.5	1	2	1.0	0.7	1	0.5	2	VI
0.2	0.5	1	2	0.2	7.0	1	0.5	2	VII
0.2	0.5	1	2	0.2	0.7	5	0.5	2	VIII
0.2	0.5	1	2	0.2	0.7	1	5.0	2	IX
0.2	0.5	1	2	0.2	0.7	1	0.5	3	X

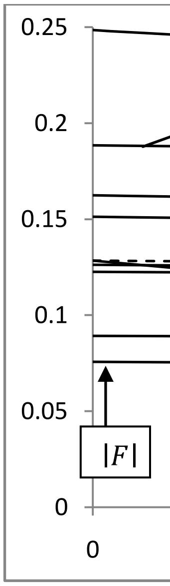


Fig.3. Amplitude of skin friction.

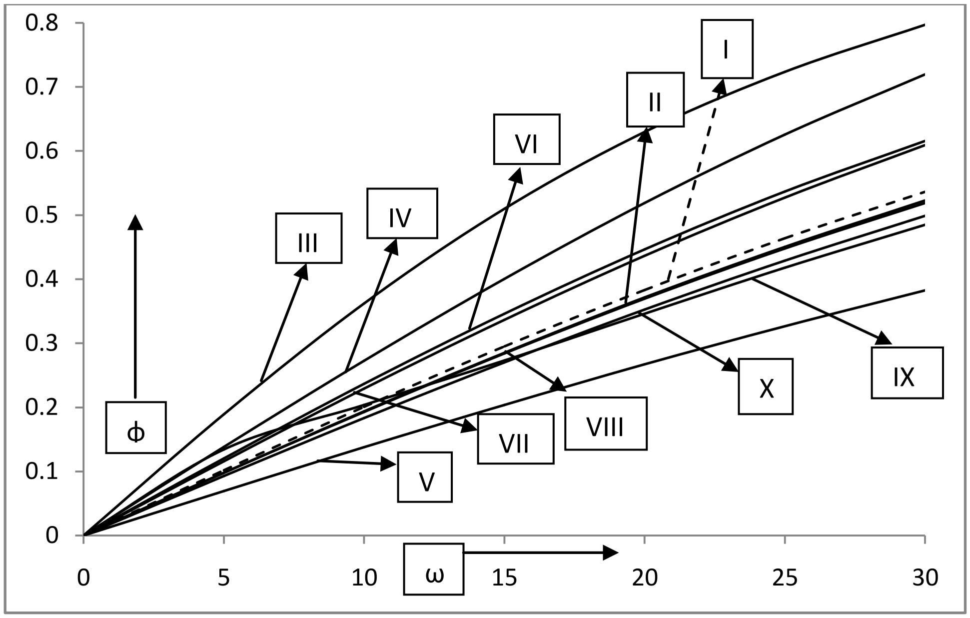
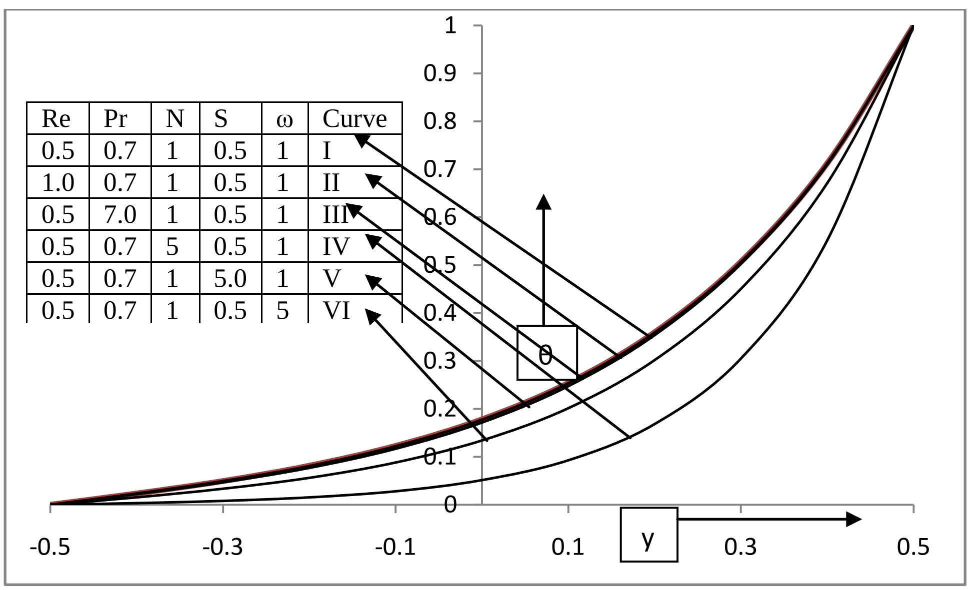


Fig.4. Phase of skin friction.



Re	Pr	N	S	ω	Curve
0.5	0.7	1	0.5	1	I
1.0	0.7	1	0.5	1	II
0.5	7.0	1	0.5	1	III
0.5	0.7	5	0.5	1	IV
0.5	0.7	1	5.0	1	V
0.5	0.7	1	0.5	5	VI

Fig.5. Temperature profiles for z=0.5 and t=π/2.

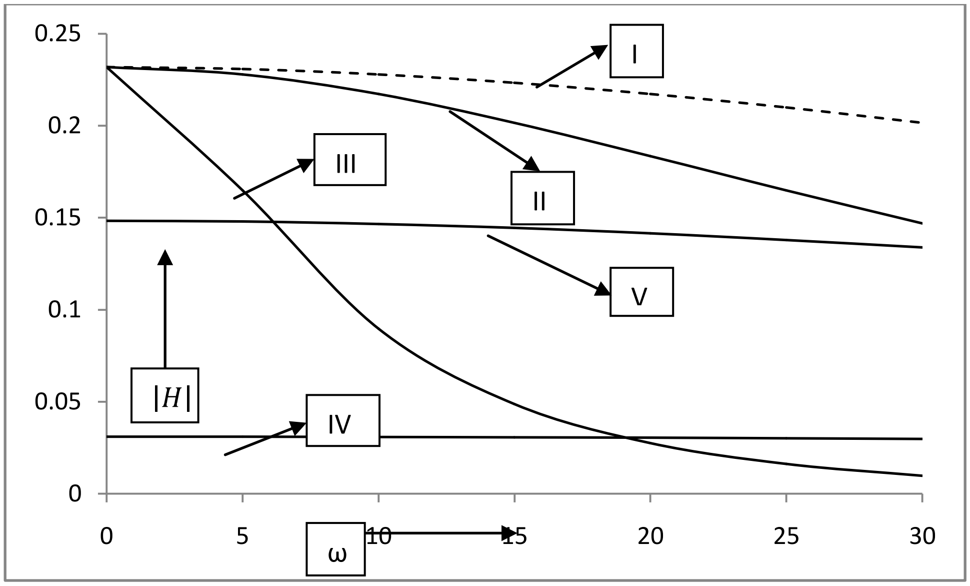


Fig.6. Amplitude of heat transfer coefficient Nu.

Table 3. Sets of parametric values plotted in Figs.6. & 7.				
Re	Pr	N	S	Curves
0.5	0.7	1	0.5	I (---)
1.0	0.7	1	0.5	II
0.5	7.0	1	0.5	III
0.5	0.7	5	0.5	IV
0.5	0.7	1	5.0	V

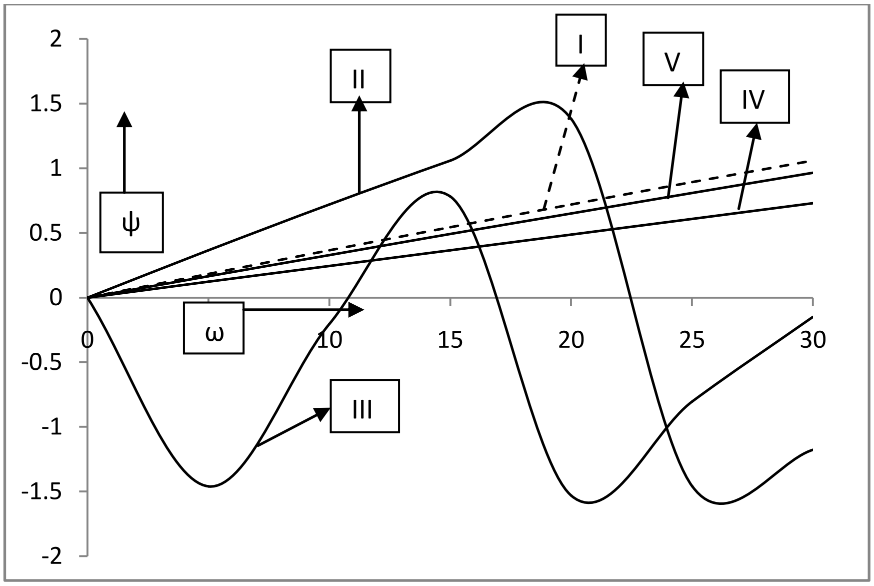


Fig.7. Phase angle ψ of heat transfer coefficient Nu.

References

1. Ahmed N, Sharma D, Deka H (2012) MHD mixed convection and mass transfer from an infinite vertical porous plate with chemical reaction in presence of heat source. Applied Mathematical Sciences 6: 1011-1020.
2. Fedorov A G, Lee K H, Viskanta R (1998) Inverse optimal design of the radiant heating in materials processing and manufacturing. J Materials Engineering and Performance 7: 719-726.
3. Garg B P (2012) Combined effect of thermal radiations and hall current on vertically porous plate in a rotating system with variable temperature. Int Jou Pure and App Math 81: 335-345.
4. Garg B P (2013) Magnetohydronamics and radiation effects on the flow due to moving vertical porous plate with variable temperature. Proc Natn Acad Sci 83: 327-331.
5. Hamza M M, Isah B Y, Usman H (2011) Unsteady heat transfer to MHD oscillatory flow through a porous medium under slip condition. Int J Computer Application 33: 12-17.
6. Jain N C, Gupta Poonam (2007) Unsteady magnetopolar free convection flow in slip flow regime with variable permeability and constant heat flux. J Energy Heat and Mass Transfer 29: 227-240.
7. Jain N C, Vijay, Kumar Dinesh (2012) Three dimensional free convective and mass transfer slip flow in a porous medium with heat source. J Energy Heat and Mass Transfer 34: 315-331.
8. Khaled A R A, Vafai K (2004) The effect of the slip condition on stokes and couette flows due to an oscillatory wall exact solution. Int J Nonlinear Mech 39: 795-809.
9. Kumar Anil, Varshney C L, Sajjan Lal (2010) perturbation technique to unsteady MHD periodic flow of viscous fluid through a planner channel. J Engineering and Tech Research 2: 73-81.

10. Kumar R, Singh K D (2011) Unsteady MHD flow of radiating and reacting fluid past a vertical porous plate with cosinusoidally fluctuating temperature. International J Appl Math and Mech 7: 19-35.
11. Lentes F T, Siedow N (1999) Three-dimensional radiative heat transfer in glass cooling processes. Glass sci Technology Glastechnische Berichte 72: 188-196.
12. Malikov G K, Lovanov D L, Malikov K Y, Lisienko G V, Viskanta R, Fedorov A G (2001) Direct flame impingement heating for rapid thermal materials processing. Int J Heat and mass Transfer 44: 1751-1758.
13. Makinde O D, Mhone P Y (2005) Heat transfer to MHD oscillatory flow in a channel filled with porous medium. Rom Journ Phys 50: 931-938.
14. Mehmood A, Ali A (2007) The effect of slip condition on unsteady MHD oscillatory flow of a viscous fluid in a planer channel. Rom Journ Phys 52: 85-91.
15. Morques Jr W, Kermer G M, Shapiro F M (2000) Couette flow with slip and jump boundary conditions. Continuum Mech Therodynam 12: 379-386.
16. Raptis A A, Perdikis C, Leontitsis K (2003) Effects of radiation in an optically thin gray gas flowing past a vertical infinite plate in the presence of a magnetic field. Heat and Mass Transfer 39: 771-773. doi: 10.1007/s00231-002-0317-8.
17. Rhodes C A, Rouleau W T (1966) Hydromagnetic lubrication of partial metal bearings. J Basic Eng T ASME 88: 53-60.
18. Sharma P K, Chaudhary R C (2003) Effect of variable suction on transient free convection viscous incompressible flow past a vertical plate with periodic temperature variations in slip-flow regime. Emirates Journal for Engineering Research 8: 33-38.
19. Sharma P K (2005) Fluctuating thermal and mass diffusion on unsteady free convection flow past a vertical plate in slip-flow regime. Lat Am Appl Res 35: 1-12.
20. Singh K D (1992) Unsteady free convection flow past a hot vertical porous plate with variable temperature. Proc Indian Natn Sci Acad 58: 537-544.
21. Singh K D, Chand Khem (2000) Unsteady free convective MHD flow past a vertical porous plate with variable temperature. Proc Nat Acad Sci 70: 49-58.
22. Singh K D (2011) Hydromagnetic forced convective oscillatory slip flow through porous medium in a vertical channel with thermal radiation. Proc Indian Natn Sci Acad 77: 19-30.
23. Singh K D (2012) Solution of MHD oscillatory flow through porous medium in a vertical porous channel in slip-flow regime. J Energy Heat and Mass Transfer 34: 217-232.
24. Sumathi K, Anuradha S, Arunachalam T (2011) Heat and mass transfer in an unsteady three dimensional mixed convection flow past an infinite vertical porous plate with cosinusoidally fluctuating temperature. International. J Engineering Science and Technology 3: 8569-8578.