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STUDY OF NON-LINEAR PROGRAMMING IN OPTIMIZATION THEORY CONTAINING SUPPORT FUNCTION

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ABSTRACT

Optimization is a rich and thriving mathematical discipline. Properties of minimizes and maximizes of functions rely intimately on a wealth of techniques from mathematical analysis, including tools from calculus and its generalizations, topological notions, and more geometric ideas. The theory underlying current computational optimization techniques grows ever more sophisticated—duality-based algorithms, interior point methods, and control-theoretic applications are typical examples. The powerful and elegant language of convex analysis unifies much of this theory.

KEYWORD: Optimization, Non-Linear Programming, Optimization Theory, Support Function

INTRODUCTION

Optimization theory is one of the most significant and fascinating branches of applied mathematics. It is formally concerned with the process of maximization or minimization of a desired function while satisfying the prevailing constraints. This has captured almost entire realm of human progress. In fact, nature has a galore of situations where optimum system status are generated. In metals and alloys, the atoms take positions of least energy to form unit cells. These unit cells define crystalline structure of materials. Genetic mutation for survival is another example of nature's optimization process. Like nature, human organizations have also worked hard towards finding excellence. Solutions of their problems have been sought mostly on the basis of experience and judgment. However, in the world of today, the increased competition and consumer demands often require optimum solutions rather than just feasible solutions. It has been experienced that optimization of design process saves money for a company by simply reducing the development time. Thus the theory of optimization deals with choosing the best alternative amongst several alternatives in the sense of given function with minimum possible resources. This generates a class of problems termed as mathematical programming problems. The optimum seeking methods are known as mathematical programming techniques and generally studied as a part of operations research.

Mathematical programming occupied a status of scientific field in its own right during late 1940's and since then it has undergone tremendous development. It is now considered as one of the most lively and exciting branches of modem mathematics having extensive applications in various contexts, such as, engineering, economics and natural sciences. A very common instance of a mathematical programming problem appears in finding minimum weight design of structure subject to constraints on stress and deflection.

The form of a mathematical programming problem is as follows, (MP): Optimize (minimize / maximize) f(x). subject to

gi(x)< 0, i = 1, 2,3…….

hj(x) = 0,j = 1,2,3......k,

X e X

Here the flinction f and each fj and hj are real valued functions defined on ndimensional Euclidean space R" and X c R". This is referred to as the general mathematical programming problem. The constraints, gi(x) < 0, i = 1, 2, m are referred to as to as inequality constraints, the constraints hj(x) = 0, j = 1, 2, ..., k are called equality constraints. The inclusion x e X is known as an abstract constraints. If the objective and constraint functions are differentiable then we describe the above problem as differentiable program. If the objective and the inequality constraints are affine function and X is a convex set, then the above problem is known as a convex programming problem.

NONLINEAR PROGRAMMING CONTAINING SUPPORT FUNCTIONS

In mathematical programming, there are numerous research articles that have discussed duality theory extensively for a program containing the square root of a positive semidefmite quadratic function by several authors, e.g., Chandra et.al [27], Zhang and Mend [112] and the references cited there. The popularity of this kind of problems lies in the fact that, even though the objective and constraint functions are nondifTerentiable, a simple formulation of the dual may be given. Nonsmooth mathematical programming theory deals with much more general type of functions that require generalized subdifferentials [84] or quasidifferentials [42]. However, the square root of a positive semidefinite quadratic form is one of few cases of a nondifTerentiable function whose subdifTerenital or quasidifferential can be written explicitly. Here a term with square root of a positive semidefinite quadratic form is replaced by a somewhat more general function, namely, the support function of a compact convex set, the subdifferential can be simply expressed.

NON-LINEAR PROGRAMMING PROBLEM

The Linear Programming Problem which can be review as to M aximize Z = Xn j=1 cjxj subject to Xn j=1 aijxj ≤ bi for i = 1, 2, ..., m and xj ≥ 0 for j = 1, 2, ..., m The term ’non linear programming’ usually refers to the problem in which the objective function (1) becomes non-linear, or one or more of the constraint inequalities (2) have non-linear or both. Ex. Consider the following problem M aximize(M inimize) Z = x 2 1 + x 2 2 + x 3 3 subject to x1 + x2 + x3 = 4 and x1 , x2 , x3 > 0

Graphical Solution

In a linear programming, the optimal solution was usually obtained at one of the extreme points of the convex region generated by the constraints and the objective function of the problem. But, it is not necessary to find the solution at extreme points of the feasible region of non-linear programming problem. Here, we take an example below :- Example 1. Solve graphically the following problem:

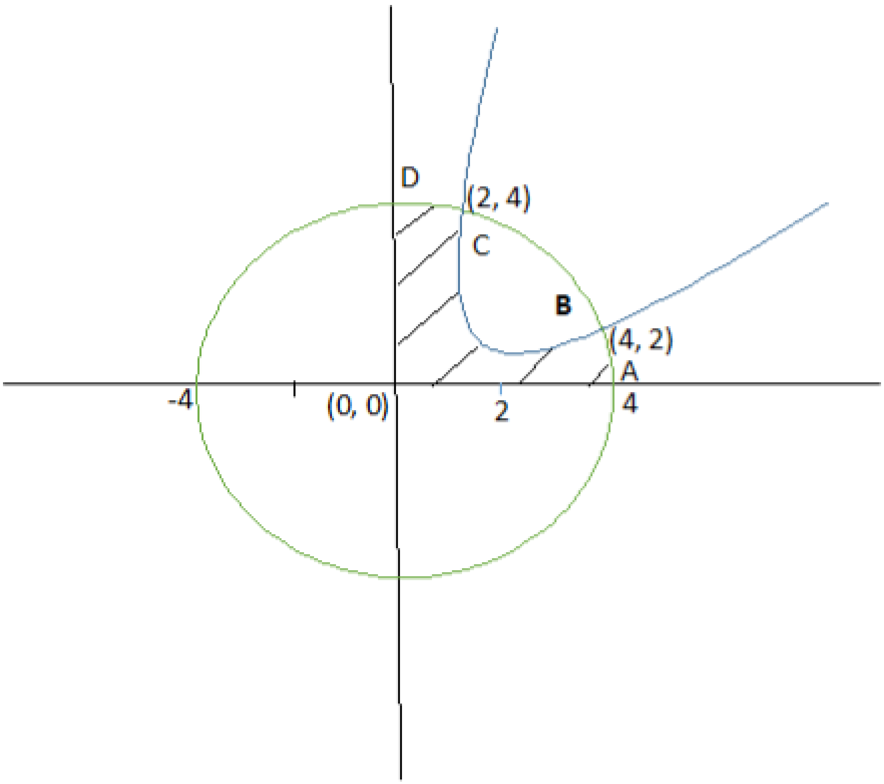
Maximize Z = 2x1 + 3x2

subject to x2 1 + x 2 2 6 20,

x1x2 6 8 and x1, x2 > 0

Solution:

In this problem objective function is linear and the constraints are non-linear. x 2 1 + x 2 2 = 20 represents circle and x1x2 = 0 represents hyperbola. Asymptotes are represented by X − axis and Y − axis . Solving eqn (2) and (3) , we get x1 = −2, −4, 2, 4 . But x1 = −2, −4 are impossible (x1 > 0) Take x1 = 2 and 4 in eqn (2) and (3) , then we get x2 = 4 and 2 respectively. So, the points are (2, 4) or (4, 2). Shaded non-convex region of OABCD is called the feasible region. Now, we maximize the objective function i.e 2x1 + 3x2 = K lines for different constant values of K and stop the process when a line touches the extreme boundary point of the feasible region for some value of K . At (2, 4) , K = 16 which touches the extreme boundary point. We have boundary point of like (0, 0), (0, 4), (2, 4), (4, 2), (4, 0). Where the value of Z is maximum at point (2, 4).



∴ M ax. Z = 16

Single-Variable Optimization

A one-variable, unconstrained nonlinear program has the form

Maximize(Minimize) Z = f(x)

where f(x) is a nonlinear function of the single variable x, and the search for the optimum is conducted over the infinite interval. If the search is restricted to a finite subinterval [a, b] ,then the problem becomes

Maximize (Minimize) Z = f(x)

subject to a 6 x 6 b

some result

(1) If f(x) is continuous in the closed and bounded interval [a,b], then f(x) has global

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optima (both a maximum and minimum) on this interval.

(2) If f(x) has a local optimum at x0 and if f(x) is differentiable on a small interval centered at x0, then f 0 (x0) = 0

Two search-methods to find the optimization in one dimension

Bisection

Assume concave f(x) → all we need to find is the turning point. Steps:

1) Initially search points x1, x2, ...

2) Keep most interior point with f 0 (x) < 0 and most interior point with f 0 (x) > 0

3) Pick a point half way in between them and: if f 0 (xk+1) < 0 −→replace xmax if f 0 (xk+1) > 0 −→replace xmin

4) Repeat until desired resolution is obtained. Stopping condition:|f 0 (xk+1)| 6 Only checking if positive or negative ⇒ Values are ignored.

Advantages: Known no. of steps until we reach the end.

Disadvantages: Doesn't use all available information. Doesn't take into account slope and curvature.

Newtons Method

This method uses information on the curvature of the function but we need to be able to calculate the curvature in order for it to be feasible. By Taylors rule

$$f(x_i) = f(x_i) + (x_{i+1} + x_i)f'(x) + \frac{(x_{i+1} + x_i)^2}{2} f''(x) + \ldots$$

If we maximize this approximation we use both the first and second derivative information to make a guesses as to the next point to evaluate:

$$x_{i+1} = x_i - \frac{f'(x)}{f''(x)}$$

In one dimension:

F' (x) = 0 is necessary for a maximum or minimum.

f "(x) > 0 is necessary for a minimum.

f "(x) 6 0 is necessary for a maximum.

For strict inequality for this to be a sufficient condition.i.e.f' (x) = 0

and f "(x) > 0 is sufficient to know that x is a minimum.

CONCLUSION

A general nonlinear programming problem. Nonlinear programming problem is a direct extension of linear programming where we replace linear model functions by nonlinear ones. In many real-life problems, the objective function may be nonlinear but the set of constraints may be linear or nonlinear and the most general class of optimization problems is the class of problems where both the objective function and the constraints are nonlinear. These problems can be solved by using a variety of methods such as penalty and barrier methods, gradient projection methods and sequential quadratic programming (SQP) methods [25]. Nonlinear programming is "hard", because there does not exist an algorithm that can solve every NLP problem efficiently in practice then, we apply the stochastic search algorithm to find the near-optimal solution of general nonlinear programming problem.

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