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Practically analyzed: the ideals case in Ring Theory

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Abstract: Hausberger (2016) stressed the importance of the study of structuralist praxeology. We demonstrate it with this correspondence regarding Ring theory's principles. We present an analysis of a proof taken from an Abstract Algebra textbook which shows that structuralistpraxeologies intertwine algebraic, set-theoretical and logical praxeologies. It exposes a latent mystery that could significantly obstruct the promise of the self-study novel.

Keywords: Mathematical structuralism, structuralistpraxeologies, ideals in Ring Theory, Logic.

Introduction:

Hausberger (2016) stressed the importance of realistic analyses in the didactic analysis of the teaching and learning of Abstract algebra at the university level with the introductions of the principle of structuralist praxeology. His thesis is focused on epistemological studies on algebraic structuralism which has shown that in Abstract Algebra mathematical practise can be seen as applying the axiomatic method, and mathematicians can use constructs as means of presenting proof of claims on subjects. A process is a collection of techniques in the philosophy of Anthropology (Bosch &Gascon, 2014).ATD provides the basic model for representing each human operation by quadruples, defined as praxeologies relating to the institutions it establishes: these combine the praxis (the T-taking of a kind and the technological methods of a set of) with a logus that contains two layers of definition and a rationale for the praxis: technology − and philosophy −. Hausberger (2016) thought that by clarifying the structuralistic approaches activities in Abstract Algebra would enlighten and make their reasoning more apparent. In order to provide new light to Abstract Algebra's transformation problem (Hausberger, 2013), the development of a functional model of reference will direct more didactical interventions. A new approach to the topic of abstract algebra.

Hausberger (2016) identified typical tasks and strategies in the arithmetic of abstract rings and researched structuralistpraxeologies built for students on an online mathematical platform. theory of Euclidian, the major fields of ideal and special factorisation. The analytical results here was, instead, an extract from training teachers in a textbook's approach to a noetherian ring exercise. The notion of ideal, which is currently studied by the scholar, is the core mathematical term. By studying this example in depth, we shall establish the statement that structuralistpraxeologies entail interplaying algebraic, conclusive and logical praxeologies, thereby unveiling a latent ambiguity that could seriously inhibit the ability of the self-study book.

Structuralistpraxeologies as algebraic, theoretical and logical interconnectors

The definition in structuralistpractice

The by-products of the full overhaul of the classical algebra by the Noether's School in the 1920s, are structural techniques (Hausberger, 2013 & 2016). They are based on the structuralist constructs that are now standard: sub-structures, homomorphisms, isomorphist theorems, products or quantities, quotients, etc. Hausberger (2016)emphasised that typical tasks in Abstract Algebra frequently can be overcome by means of simple techniques. If the logos block includes a structural theorem, praxeology may be considered structuralist. Nonetheless, it is possible to detect a gradation of its structuralistic dimension (OC). Structuralistpraxeologies reflect the real and abstract dialectics used in Abstract Algebra: work involving concrete structures and individual objects is completed using abstract and general structural considerations. In the sequel, examples are given.Structuralistpractise is distinguished by the fact that sub-praxeologies of an algebraic or set theoretical or logical nature are frequently studied in this paper.

Praxeologies Algebraic and analytical

Noether defined her own work, following Dedekind, as "theoretical basis of algebra" (Hausberger, 2013). From the epistemological point of view, the transformation from the thought of elements to the thought of the chosen sub-sets and homomorphisms is distinctive. The distinctive subcategories are the centre of homomorphism, which is why in group theory the standard subcategories and the ideals of ring theory. The value of chain conditions on ideals leading to the description of Noetherian rings was discovered (see below). In other words, set theoretical ideal operations are linked with algebraic element properties. We will present this relation via a "dictorial" below. It illustrates the mixture of algebraic praxeologies (at the unit level) and set-theoretical praxeologies (at the structure level), but it also contributes to the implementation of logical praxeologies, in particular to descend from the principles to the entitled elements.

Praxeologieslogic

Many activities require justification and argument in Abstract Algebra and thus logical praxeologies. Durand-Guerrier(2008) has shown that Copi (1954) 's natural deduction is an effective method for evaluating and tracking mathematical facts. It makes the recognition of certain phases in which mathematical argument is silenced and supports the assertion that the data is closely connected between mathematics and logic. We would use Copi's natural deduction to define the philosophical praxeologies likely to occur in facts and proof: exclusion and inference implementation, universal quantifiers and quantifiers of existences, constraint of the quantifying domain. Theory is the first-order logic, and technology is logical (e.g. assertions valid in all non-empty domain interpretations).In the natural deduction of Copi, a general non-emptious universe is explored and certain aspects involve realistic supervision, as shown below. The table below summarises typical logical praxeologies which may be involved in facts and thus in the analysis of structural praxeology.

Triplets (type of activities, infrastructure and technology) are provided:

index	Type of tasks	Technique	Technology	Example of use
L1	Elimination of an implication	Asserting the antecedent – asserting the consequent	$[(P \Rightarrow Q) \wedge P] \Rightarrow Q$ (tautology)	Deduction based on a conditional theorem
L2	Introduction of implication	Recognizing that *Q* has been proved under the hypothesis *P*, and assert "*P* ⇒ *Q*"	$\neg(P \wedge \neg Q) \Leftrightarrow (P \Rightarrow Q)$	Conclusion of the proof of a conditional statement
L3	Elimination of a universal quantifier	Deleting the quantifier, introducing of a generic element of the universe, assigning this element to every occurrence ofthe variable in the open statement.	$[\forall x\, (F(x)] \Rightarrow F(y)$	Using a universal statement in a proof by generic element.
L4	Introduction of a universal quantifier	Given a true statement involving a generic element of a domain *U*, assert the corresponding universal statement	No logical theorem. Need to control that the element is actually a generic element of *U* (no other assumption on this element hasbeen done)	Conclusion of proof by generic element.

L5	Introduction of an existential statement	Givenanelementoftheuniverse *U* satisfying an opensentence, assert that the corresponding existential statement istrue.	$F(y) \Rightarrow \exists x\, F(x)$	Conclusion of the proof of an existentialst atement.
L6	Elimination of an existential statement	Given a true existential statement, introduce an element satisfying the corresponding opensentence.	No logical theorem. Need to control that the name of the element has not been used prior in the proof	Initiating a proof by generic element
L7	Restriction of the domain of quantificatio n	Given a universal statement true in a domain *A*, assert it on a subdomain *B* of*A*.	$[(\forall x\, (A(x) \wedge F(x)) \wedge (\forall x\, (B(x) \Rightarrow A(x))] \Rightarrow [(\forall x\, (B(x) \wedge F(x))$	Fitting the statement with the antecedent of aconditional statement
L8	Transformati o n of a statement preserving its truth value	Substitute an equivalent statement to a given statement	In the case of implication: $[(\forall x\, (P(x) \Rightarrow R(x)) \wedge (\forall x\, (Q(x) \Leftrightarrow R(x))] \Rightarrow [(\forall x\, (P(x) \Rightarrow Q(x))$	Using the dictionary of p roperties elements/st ructures (*cf.* table2)

Figure 1: Set of conceptual prerequisites through copy

The case of Ring Theory Ideals

Ideal and ecological design

An ideal I of an additive group (A, +, ±) is, by definition, the sub-group of the additive group (A,+); ii) if an os A and x os I, ± x os I is an additive group. The first author underwent an epistemological and didactic analysis of the philosophy of the ideal to investigate ecology in French university education, including its ecosystems and niches (Artaud, 1997). This epistemological research began with Kumer developing the ideal number in 1847 and enhanced the rise of abstraction which led to the term we use today (Jovignot, to show) in the 1920s through the work of Noether.

As for ecology of the ideal, the epistemologic analysis entitled "General ring theorems" (quotient rings and isomorphism theorems), abstract arithmetic, and exclusion theory

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"provided for the discovery of the following primary ecosystems. Based on those observations, Jovigno created an empirical method in the algebra textbooks for undergraduates and Masters students to define environments and niches in an ideal concept. A first review of 3 textbooks led to an increase in this grid, which was then extended for a survey of seven French textbooks which were considered to reflect the ecology of the ideal definition and its use in the various universities where it is taught in France.This research verified that the general Ring Theory and the arithmetic in abstract rings are the main habitats in the conception of the ideal. It also allowed habitats not previously identified to be presented, such as modular theory and algebraic geometry. Lastly, only a specialised machine algebra handbook existed in our sample exclusion theory.

Ideals and structural experience in the abstract ring arithmetic

The mathematical arithmetic of abstract rings in "Russian dolls" is defined as a mathematical structure: Euclidean, major ideal fields (PID) and special fields of factorisation (UFDs), generalising the properties of the ring of integral elements and mathematical theorems which vary from the former to the latter class. Popular tasks are to prove that a ring like Gauss's Z[i] integer ring, for example, is one type or the other. More abstract activities, such as the one analysed below, require new interactions between these groups. The notion of ideal is the central one. In fact, the class can be explicitly defined by a property on ideals (such as PIDs) or by properties on elements (such as UFDs) that may be connected to ideals with the next "dictionary," as previously stated. The dictionary is useful for interpreting the praxeology used in the later learned mission.

index	Conditions of validity	Level of elements	Level of structures
D1		a divides b	(a) contains (b)
D2		a and b are associates	$(a) = (b)$
D3	$p \neq 0$	p is a prime element	(p) is a prime ideal
D4	*A* is a principal ideal domain	p is irreducible in *A*	(p) is a maximal ideal of *A*
D5	A is a unique factorisation domain	*d* is a *gcd*of *a* and *b*	$(d) = (a) + (b)$

Figure 2: Element / structure patented dictionary

The mission at hand

The following section will provide a realistic study of exercise concerning the perfect principle taken out of a book for MA students in preparation for the Agrégation1 of the French: Francinou, S. In the following section we provide. Gianella, H. &Gianella [2004]. This book is used extensively in France by university students. The authors evaluated and presented evidence of classical exercises in algebra. In the selected exercise, students are asked to create a relation between Noetherian integral domains and PIDs.

The practise (our translation) is as follows:

Let A be an integral field of Noetheria. We assume that any ideal limit of A is primary.

1) Show that the Domain A is a single factor.

2) Show that every non-zero prime ideal shall be complete, principal and p where p is irreducible.

3) View A's the main ideal realm. (a.k.a. p.57)

Our analysis will be restricted to question 1. The authors propose the following classical parameters in which E defines the function of factorization life and U the function of unicity:

A is a UFD only if and when:

a) every chain (a1) < (a2) < (a3) < ... increasing with principal ideals (equivalent to E) is stationary

(b) each element is primary, irreducible (U-equivalent)

The author's evidence is as follows (our French translation):

A satisfies (E) since A is noetherian. In order to determine that A is a unique factorization field, it must be proven that the ideal (p) is primitive if p is irreducible. Find a maximum container of M ideal (p).

Task research for operational reasons

Supplementing the proof

Reading the authors' proof, several measures tend to be tacit. Therefore, we have supplemented the data to research the entire collection of praxeologies. The evidence is considered complete because all conclusions are derived from natural results or standard theorems defined in Abstract Algebra. The steps of the facts in the book are numerical, and our supplements occur in italic, and where many steps are involved, are designated with letters. The supplementary proof reads:

1. A satisfies (E) since A is noetherian.

a. In truth, A is Noetherian, so by definition any increasing ideal chain is stationary.

b. Particularly any rising chain of major ideals is stationary.

c. A thus fulfills(E) by the criterion.

2. For A to be a UFD, it suffices to show that the ideal (p) is primary while p is irreducible.

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a. In fact, any irreducible element must be shown to be prime (criteria b)

b. And "p is the first" equals "(p) is the first"

c. We are simply going to demonstrate that (p) is limit. Any maximum ideal in a ring is appropriate.

3. Let p be a maximum optimal component of A and M (p).

a. We're done if there's no irreducible elements. Currently, when A is noetherian, irreducible elements occur only when A is a field.

b. p isn't a unit, so p is right and M occurs in accordance with the theorem of Krull.

4. M is main by hypothesis. Let's get a M generator.

5. So a splits p.

a. Indeed (p) is in M and M = (a), then (p) is in (a).

b. And (a) is (a) used only if a split p.

6. Because a does not constitute one unit (because M <> A), p and an are associates-in fact, a is so b in A that p = ab exists; in addition, p is so irreducible because an is not a unit, b must be a unit and a must be associates-

7. And (p) = M is maximum since only when and when their generators are associates, two principal ideals can be equal.

8. (p) is primary in particular.

Conclusion: The difficulty that the writers of the evidence appear to underestimate in our praxis review has illustrated. These complexities are primarily attributed to the disassembly of structuralistpraxeologies and their association with logical and algebraic praxeologies into many structuralistsubpraxeologies. This are fundamental for the functional application of structuralist technology. Sketchy data limiting itself to structural measures can therefore seem to be quite insufficient to self-learn students who are not familiar with the structuralist approach, although it is viewed by mathematicians as a straightforward and synthetic account. In other words, our analysis helps crack the 'transparency myth' behind facts in Abstract Algebra.

We want to document, evaluate and recreate the work of students who try or write facts from scratch. Our praxeological analysis will act as a first analysis in this upcoming analytical research. It may also be used as a starting point for the planning and execution of semi formal interviews for the students and other forms of didactical intervention. More broadly, a greater interpretation of structuralist praxeology with a view to improving didactic methods devoted to the teaching of structuralist ideas and in particular to the ideal concept is anticipated from these functional analyses, carried out on a broader basis.

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