AIRO JOURNALS
Peer Reviewed Multidisciplinary International
ISSN:2320-3714
Volume:2 Issue:3
June 2024
Impact Factor:10.2
Subject:Computer Science

HEAVY TRAFFIC ANALYSIS OF DATA TRANSMISSION AND COMMUNICATION SYSTEMS USING QUEUING THEORY

Karti Garg
Research Scholar

Abstract

Introduction: First of all, managing large heavy traffic is essential for maintaining performance and dependability in the field of data transmission and communication networks. This study investigates the use of queuing theory to analyse heavy traffic, providing a methodical way to anticipate and resolve congestion problems.

Aim: The purpose of this study is to explore and use Queuing Theory models, namely (M/M/1): ((C+1)/FCFS) and (M/M/2): ((C+1)/FCFS), to the prediction and stabilisation of congestion rates in communication and data transmission systems during periods of heavy traffic.

Method: To analyse heavy traffic, we use Queuing Theory models (M/M/1) and (M/M/2) with ((C+1)/FCFS) configurations. The main goals of the research are to develop methods for predicting periods of heavy traffic and creating consistent congestion rate formulas. To provide a thorough strategy for real-time traffic estimation and monitoring, these models are combined with standard heavy traffic monitoring characteristics.

Findings: Using Queuing Theory models, the study presents efficient approaches for predicting periods of heavy traffic and creating reliable equations for the rate of congestion. The study demonstrates how well (M/M/1) and (M/M/2) models forecast and control congestion under various traffic loads.

Conclusion: Performance and resilience are improved when Queuing Theory models are incorporated into the analysis of heavy traffic in data transmission and communication systems.

The established techniques allow for precise forecasts and effective data flow management, guaranteeing that systems can withstand spikes in traffic volumes with little disturbance.

Keywords: Queuing Theory, Heavy traffic, Data transmission, Communication, Traffic monitoring, Server.

1. INTRODUCTION

In the ever changing world of digital communication, data transmission systems' performance and efficiency are critical. The growth of internet-connected devices, streaming services, and cloud computing has resulted in an increase in the volume of data traffic. Therefore, it is imperative to develop effective approaches for analysing and managing heavy traffic [1]. This study applies the mathematical framework of queuing theory, which offers important insights into the behaviour of lineups in many service systems, to the heavy traffic analysis of data transmission and communication networks.

Based on the study of waiting lines, queuing theory provides a strong toolkit for simulating and examining systems with high levels of delay and congestion. Queues are the buffers in network devices that hold data packets as they wait to be processed in the context of data communication [2]. It is crucial to comprehend these queue dynamics, particularly in instances of high traffic, in order to maximise network efficiency, lower latency, and avoid packet loss [3]. We may create prediction models using Queuing Theory that assist network administrators in anticipating congestion and putting procedures into place to preserve system stability.

Two basic queuing models are used in this study: (M/M/1): ((C+1)/FCFS) and (M/M/2): ((C+1)/FCFS. These models are particularly useful for studying communication systems because of their analytical tractability and relevance to actual network conditions. They are distinguished by their exponential inter-arrival and service time distributions. The (M/M/2) model provides a comparative view of how additional resources affect congestion and performance by extending the single-server queue model of the (M/M/1) model to a dual-server scenario.

Our research attempts to extract useful techniques for predicting high traffic and creating reliable congestion rate equations. We develop a comprehensive framework for real-time traffic estimation and monitoring by combining these queuing models with conventional heavy

ISSN:2320-3714
Volume:2 Issue:3
June 2024
Impact Factor:10.2
Subject:Computer Science

traffic monitoring characteristics. This methodology not only improves the precision of traffic forecasts but also facilitates the creation of preemptive strategies to efficiently handle heavy traffic.

The design and administration of contemporary communication systems will be significantly impacted by the findings of this study. With the increasing complexity of networks and the growing need for dependable, high-speed data transmission, the capacity to anticipate and manage traffic congestion is essential for maintaining both overall user satisfaction and quality of service (QoS). Furthermore, by applying the lessons learned from Queuing Theory, more robust and scalable network infrastructures that can adjust to changing needs can be built.

By bridging the gap between theoretical queuing models and real-world network management, this research offers a solid framework for analysing heavy traffic in communication and data transmission networks. We provide innovative answers to the problems brought on by growing data traffic by utilising queuing theory, which advances the development of dependable and effective communication networks.

2. LITERATURE REVIEW

Ata and Peng (2018) [4] discuss the difficulties in managing multiclass queueing systems in situations of high traffic when model parameters are unclear. The precise parameters of the queueing model may not be known in many real-world situations, which make it challenging to use conventional control strategies successfully. Asymptotic analysis approaches, which Cohen's work offers, can be used to create robust control strategies that function well even in the presence of substantial uncertainty regarding the system characteristics. The study shows that near-optimal performance may be attained with these strategies, improving the efficiency and dependability of multiclass queueing systems in unpredictable scenarios.

Cohen (2019) [5] focuses on the difficulties in managing multiclass queueing systems in situations of high traffic when model parameters are unclear. It can be challenging to implement conventional control rules successfully in many real-world situations since the precise parameters of the queueing model may not be known. Cohen's research offers asymptotic analytic methods for creating reliable control strategies that function effectively even in the presence of high system parameter uncertainty. The study shows that these methods

AIRO JOURNALS
Peer Reviewed Multidisciplinary International
ISSN:2320-3714
Volume:2 Issue:3
June 2024
Impact Factor:10.2
Subject:Computer Science

can be used to attain near-optimal performance, improving the efficiency and dependability of multiclass queueing systems in unpredictable scenarios.

Gurvich (2014) [6] looks at, with an emphasis on queue-proportion trains, the suitability of heavy-traffic consistent state approximations in multiclass queueing networks. Arrangements known as queue-proportion disciplines alter administration rates as per the proportions of different classes' queue lengths. Gurvich's work demonstrates that reliable performance forecasts for these systems can be obtained using heavy-traffic steady-state approximations. The study shows that these approximations hold under more general conditions than previously thought, which extends their applicability to more real-world queueing scenarios. The theoretical groundwork for the study and design of multiclass queueing networks utilising heavy-traffic approximations is provided by this work.

Sani and Daman (2014) [7] offer a fundamental method for mathematical modelling of systems with high traffic queues. The significance of creating reliable models that can precisely forecast system performance in situations with high traffic is highlighted by their work. They investigate several mathematical methods for simulating queue dynamics and offer fixes for typical issues that arise in highly trafficked systems. grasp the fundamental ideas and procedures that support heavy traffic queueing theory requires a thorough grasp of this study. The writers go over several modelling techniques, stressing the difficulties and possible fixes in efficiently managing lines when they are almost full.

Izagirre, Verloop, and Ayesta (2015) [8] examine how non-preemptive multiclass queues with relative priorities behave under high traffic conditions. Their research examines systems in which various client classes are given varied priority levels; however, preemption is prohibited, meaning that once a service starts, it cannot be stopped. The authors show how relative priorities affect system behaviour and derive performance indicators using heavy-traffic analysis. The trade-offs of various priority schemes are emphasised in this study, which also offers helpful advice for creating queueing systems that must strike a balance between efficiency and justice for various client classes.

3. THE MATHEMATICAL MODEL OF THE QUEUING THEORY

ISSN:2320-3714
Volume:2 Issue:3
June 2024
Impact Factor:10.2
Subject:Computer Science

The framework utilised to examine and forecast the behaviour of waiting lines or lineups is known as the mathematical model of queuing theory. Arrivals, service times, queue discipline, and server count are some of its essential elements. Customers enter the system through the arrival process, which is represented by arrivals and follows a predetermined arrival pattern (such as deterministic or Poisson) [9]. After a customer reaches the head of the queue, their service time is determined by how long it takes to serve them; this is frequently represented using exponential or other distributions. Rules like first-come, first-served, or priority-based that determine which client is serviced next are referred to as queue discipline [10]. How many service points are available to accommodate consumers concurrently is indicated by the number of servers. Operational analysis techniques, Markov processes, and probability distributions are used to mathematically characterise these components and yield performance measurements such as average waiting time, queue length, and server utilisation [11]. Applications of queuing theory are seen in a number of industries, such as manufacturing, telecommunications, healthcare, and transportation, where it is used to maximise system efficiency, reduce waiting times, and optimise resource allocation [12].

When heavy traffic analysis is applied to communication and data transmission networks in queuing theory, the mathematical model becomes especially concentrated on situations in which the system is highly utilised and congested [13]. Here, the amount of servers or processing nodes available, the arrival rates of data packets or requests, and the service times for processing these packets are all taken into account by the queuing model. Arrival rates can exhibit patterns similar to those seen in heavy traffic; these patterns are frequently represented as Poisson processes or more intricate arrival processes that represent spikes in activity [14]. Depending on the features of the system, service durations are sometimes modelled using distributions like exponential or heavy-tailed distributions, which account for the time needed to process a packet or handle a communication request.

When several things compete for limited processing resources, queue discipline becomes important in deciding how packets or requests are managed, influencing factors like fairness, prioritisation, or quality of service guarantees [15]. The quantity of servers indicates the system's ability to process several requests at once, which affects latency, throughput, and other performance indicators. Stochastic processes, Markov chains, and sophisticated probability theory are some of the mathematical techniques used to analyse these systems in heavy traffic

AIRO JOURNALS
Peer Reviewed Multidisciplinary International
ISSN:2320-3714
Volume:2 Issue:3
June 2024
Impact Factor:10.2
Subject:Computer Science

situations. Queue lengths, waiting times, server utilisation rates, and throughput capacity are some of the performance metrics that are obtained and are essential for streamlining traffic management plans, resource allocation, and network architecture [16]. Therefore, queuing theory is essential to building reliable and effective communication networks that can manage high-volume data transfer demands.

A basic queuing model can be found in all of the processes involved in network communication, from sending and receiving data to coding, decoding, and sending the data to a higher layer. This corresponding technique can be abstracted as a Queuing theory model, as shown in figure 1, according to Queuing Theory. Given that this type of straightforward data transmission system complies with the queue model.

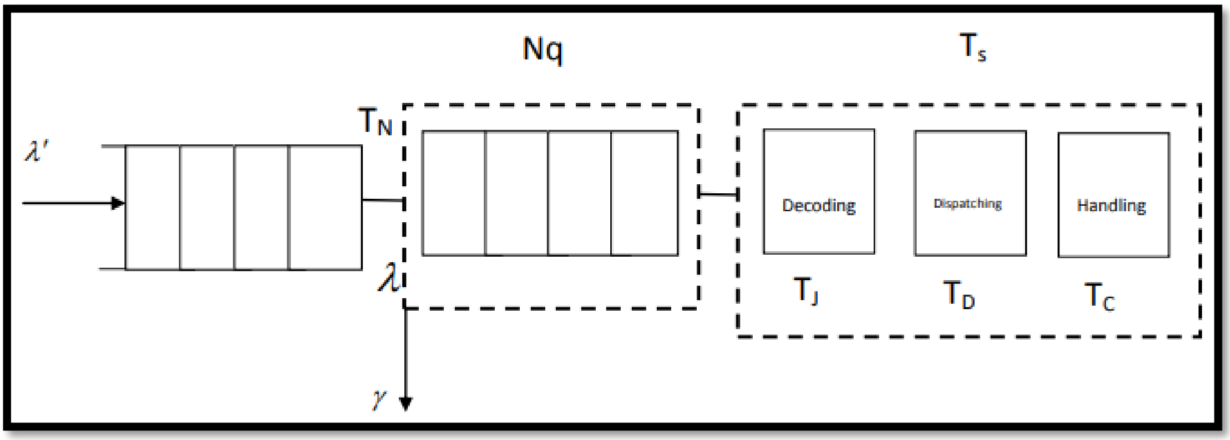

Figure 1: Queuing Theory Model of the Communication Process

From the above figure-1:

λ': The sender's rate of transmission.

T_N: Time of transportation delay.

λ: The speed at which data packets arrive.

N_q: The number of data packets kept in the buffer for short-term storage.

γ:The rate of packets that the recipient sends incorrectly, or the receiver's loss rate.

T_s: The server's data packet service time

Peer Reviewed Multidisciplinary International
ISSN:2320-3714
Volume:2 Issue:3
June 2024
Impact Factor:10.2
Subject:Computer Science

In this case, $T_s = T_J + T_D + T_C$,

T_J: Time of Decoding

T_D: Time of dispatch

T_C: Calculating time or, evaluating time or handling time.

4. MODEL 1: SINGLE-SERVER QUEUING MODEL (M/M/1) FOR HEAVY TRAFFIC ANALYSIS (C+1/FCFS)

Assuming a Poisson distribution and a Markov process, the sending process of the sender is represented by the first "M" in the M/M/1 model, while the receiving process is represented by the second "M." A single channel is indicated by the number "1". The queue length at time t can be represented as N(t)=n. As a result, the probability indicates the likelihood that the queue will have length n at time t.

$$P_n \text{ prob } (t) = [N(t) = n]$$

In this instance,

λ_n = Arrival rate into the state

μ_n =Departure rate from the initial state n.

The transition rate diagram is available.

Figure 2 shows the diagram of the transition rate. A collection of differential difference equations describes the system.

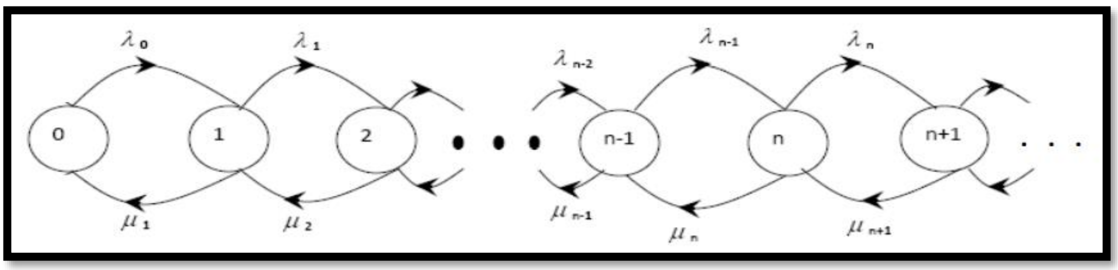

Figure 2: Diagram of the state transition

AIRO JOURNALS
Peer Reviewed Multidisciplinary International
ISSN:2320-3714
Volume:2 Issue:3
June 2024
Impact Factor:10.2
Subject:Computer Science

A differential difference equation system is.

$$\frac{d}{dt}\{P_n(t)\} = -\lambda_n P_n(t) - \mu_n P_n(t) + \lambda_{n-1} P_{n-1}(t) + \mu_{n+1} P_{n+1}(t) \quad , \text{for } n\geq 1 \qquad (1)$$

$$\text{And } \frac{d}{dt}\{P_0(t)\} = -\lambda_0 P_0(t) + \mu_1 P_1(t); \quad \text{for } \ n=0 \qquad (2)$$

For the M/M/1 model, we let

$\mu_n = \mu$ and $\lambda_n = \lambda$

Where the constants λ and μ are

Next, (1) and (2) come down to

$$\frac{d}{dt}\{P_n(t)\} = \lambda P_{n-1}(t) + \mu P_{n+1}(t) - (\lambda + \mu) P_n(t); \quad \text{for } n\geq 1 \qquad (2)$$

$$\text{And } \frac{d}{dt}\{P_0(t)\} = -\lambda P_0(t) + \mu P_1(t); \quad \text{for } \ n=0 \qquad (3)$$

In this case, the arrival rate is denoted by λ, and the service rate by μ.

When everything is in a stable state

$$\underset{t\to\infty}{\text{Lt}}\, P_n(t) = P_n$$

$$\text{And } \underset{t\to\infty}{\text{Lt}}\, \frac{d}{dt}\{P_n(t)\} = 0$$

Thus, based on (2) and (3), when $t\to\infty$ we have

ISSN:2320-3714
Volume:2 Issue:3
June 2024
Impact Factor:10.2
Subject:Computer Science

$$0 = \lambda P_{n-1} + \mu P_{n+1} - (\lambda + \mu) P_n \quad (4)$$

And $0 = -\lambda P_0 + \mu P_1$

Or, $P_1 = (\frac{\lambda}{\mu}) P_0$

When n=1 from (4), we have

$$(\lambda + \mu) P_1 = \lambda P_0 + \mu P_2$$

$$\text{or } P_2 = (\frac{\lambda}{\mu})^2 P_0$$

$$\text{In general } P_n = (\frac{\lambda}{\mu})^n P_0$$

$$\text{or, } P_n = \rho^n P_0 \text{ where } \rho = \frac{\lambda}{\mu}$$

Additionally, ρ stands for traffic intensity or server utilisation factor.

We are aware

$$\sum_{n=0}^{\infty} P_n = 1$$

Also $P_n = \rho^n P_0$

This suggests that

ISSN:2320-3714
Volume:2 Issue:3
June 2024
Impact Factor:10.2
Subject:Computer Science

$$\sum_{n=0}^{\infty} P_n = \sum_{n=0}^{\infty} \rho^n P_0$$

$$\text{or,} \quad 1 = P_0 \sum_{n=0}^{\infty} \rho^n$$

$$\text{or,} \; P_0 = 1 - \rho, \text{ where } \rho < 1$$

Hence

$$P_n = \rho^n (1-\rho) \;, \qquad n=0, 1, 2,\ldots \tag{5}$$

Assume that L represents the queue's length in a steady state scenario. The average volume of every data packet that enters the processing module and is stored in the buffer is included.

$$L = \sum_{n=0}^{\infty} nP_n = \sum_{n=1}^{\infty} n\rho^n (1-\rho)$$

$$= (1-\rho)\sum_{n=1}^{\infty} n\rho^n$$

$$\text{Hence} \qquad L = \frac{\rho}{1-\rho} \tag{6}$$

$$\text{Also} \qquad L = \frac{\lambda}{\mu - \lambda} \qquad (\text{since, } \rho = \lambda/\mu) \tag{7}$$

If N_q displays the data packets' average volume in the buffers.

$$N_q = L - \rho = \frac{\rho^2}{1-\rho} \tag{8}$$

$$\text{Also} \qquad N_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

ISSN:2320-3714
Volume:2 Issue:3
June 2024
Impact Factor:10.2
Subject:Computer Science

The parameter is introduced into the formula (8) if the processing module is thought of as a closed region.

By applying Little's law, we obtain 1/μ = server's average service time = T_s.

$$\Rightarrow \quad \rho = \lambda T_s \quad \text{and here } \lambda = \lambda' \tag{9}$$

By using (9), (8) becomes

$$N_q = \frac{\rho^2}{1-\rho}$$

$$\text{or, } (1 - \lambda T_s) N_q = (\lambda T_s)^2$$

$$\text{or, } (\lambda' T_s)^2 + \lambda T_s N_q - N_q = 0, \quad (\text{since, } \lambda = \lambda') \tag{10}$$

The aforementioned equation (10) leads us to the conclusion that the three variables Ts (service time), (sending rate), and Nq (number of data packets kept in the buffer) are crucial for assessing the transmission system's performance. By applying Queuing Theory to evaluate performance effectively in data transmission and communication systems under high traffic conditions, we may ascertain the value of the third variable by knowing any two of the other two.

5. QUEUING THEORY AND THE HEAVY TRAFFIC MONITOR

A key idea in computer science and operations research, queuing theory is especially important for comprehending and controlling situations with heavy traffic in communication networks and systems. Queuing theory is primarily concerned with the mathematical modelling and analysis of lines or queues that are waiting [17]. It offers a systematic framework for anticipating and improving the operation of systems in which entities (such requests, clients, or data packets) arrive at a service point and wait to be processed.

Queuing theory provides important insights into how networks manage throughput during peak hours and handle congestion in the context of heavy traffic monitoring. Network engineers and academics can use it to study diverse scenarios, including varied data packet arrival rates or

AIRO JOURNALS
Peer Reviewed Multidisciplinary International
ISSN:2320-3714
Volume:2 Issue:3
June 2024
Impact Factor:10.2
Subject:Computer Science

different server or router processing capacities [18]. They can estimate important performance measures like waiting times, queue lengths, and system utilisation by examining these models.

The capacity of queuing theory to provide quantitative predictions and optimise system parameters for increased dependability and efficiency is one of its main advantages in heavy traffic analysis. For example, models such as the M/M/1 and M/M/2 queuing systems can be used by academics to mimic and comprehend the behaviour of networks under heavy load conditions. These models take into account variables that affect the network's overall performance and congestion levels, such as data packet arrival rates, service times, and server availability.

The design of efficient traffic management systems is aided by queuing theory. Network managers can put policies and protocols in place that prioritise traffic, distribute resources effectively, and reduce delays or packet loss during spikes in demand by researching queueing models [19]. By cutting down on latency, this proactive strategy not only improves user experience but also makes sure the network runs within ideal performance limits.

Through the provision of analytical tools and methodologies to anticipate, analyse, and optimise system performance under various load conditions, queuing theory plays a crucial role in the heavy traffic monitoring of data transmission and communication systems [20]. When used in dynamic and demanding contexts, it enables network engineers and researchers to make well-informed decisions that improve network resilience, efficiency, and overall user pleasure.

5.1. Using Queuing Theory to Forecast Heavy Traffic

In data transmission and communication networks, heavy traffic is a common problem. In severe traffic situations, the system's performance can drastically deteriorate, causing network congestion. Monitoring and controlling traffic congestion are the subject of much research, and applying Queuing Theory to traffic rate analysis is receiving more and more attention. We frequently test the network's routers' data handling capacities in order to predict traffic rates. Think of a router that has these configuration settings:

- **Arrival rate of data flow (λ):** The speed at which information packets travel along the path.

AIRO JOURNALS
Peer Reviewed Multidisciplinary International
ISSN:2320-3714
Volume:2 Issue:3
June 2024
Impact Factor:10.2
Subject:Computer Science

- **Service rate (μ):** The average processing speed of a packet by the router, with an average processing time of $1/\mu$.
- **Buffer capacity (C):** The highest quantity of packets that the router's buffer can store.

The packet needs to be dropped (discarded) when it comes and the buffer is full. A packet must be resent if its arrival time exceeds a predetermined threshold (timeouts). Assume that $1/\mu$ represents the average waiting time for a packet.

The probability that the queue length is i at time t is defined as P_i (t). The probabilities of the queue length can be shown as:

$P(t) = (P_0 (t), P1 (t), \ldots, P_i (t)), i = 0,1, \ldots, C+1$.

The router's date groups' queuing system therefore fulfils the basic Markov Process, and using this process, we can determine the diversion strength of model 1's matrix as follows.

$$Q=\begin{pmatrix} -\lambda & \lambda & 0 & 0 & \cdots & 0 & 0 \\ \mu & -(\lambda+\mu+v) & \lambda+v & 0 & \cdots & 0 & 0 \\ 0 & \mu & -(\lambda+\mu+2v) & \lambda+2v & \cdots & 0 & 0 \\ 0 & 0 & \mu & -(\lambda+\mu+3v) & \cdots & 0 & 0 \\ 0 & 0 & 0 & \mu & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -(\lambda+\mu+Cv) & \lambda+Cv \\ 0 & 0 & 0 & 0 & \cdots & \mu & -\mu \end{pmatrix}$$

5.2. Network Congestion Rate

The rate of network congestion is constantly fluctuating. When analysing high traffic in a network monitor, the instantaneous congestion rate and the stable congestion rate are frequently utilised. The congestion rate at time t is known as the instantaneous rate A_c (t). The system length of the queue's probability distributing, or $P_{c+1}(t)$, can be solved to find the A_c (t).

Assuming that the queue length is k, let P_k (t)(K=0,1,. . .,C+1) represent the arrival probability of the queue length for the routers group at time t.

ISSN:2320-3714
Volume:2 Issue:3
June 2024
Impact Factor:10.2
Subject:Computer Science

Then, the router's date groups' queuing system fulfils the basic Markov Process. $P_k(t)$ satisfies the following system of differential equations, according to Markov Process.

Let,

$P_k(t)$ = prob { k no. of data packets present in the system in time t }

and $P_k(t+\Delta t)$ = prob { k no. of data packets present in the system in time $(t + \Delta t)$ }

Case 1:

For $k \geq 1$

$P_k(t+\Delta t)$ = Prob { k no. of data packets present in the system at time t } × prob {no data packet×no data packets arrival in time (Δt) } departure in time Δt } + Prob { (k -1) no. of data packets present in the system at time t } × prob { no data packet×prob { 1 data packet arrival in time (Δt) } departure in time Δt } + prob { (k +1) no. of data packets present in the system at time t } × prob { no data packets arrival in time (Δt) } × prob { 1 data packet departure in time Δt }+. . .

$$\Rightarrow P_k(t+\Delta t) = P_k(t)\times\{1-\lambda_k\Delta t+o(\Delta t)\}\times\{1-\mu_k\Delta t+o(\Delta t)\}$$

$$+P_{k-1}(t)\{\lambda_{k-1}\Delta t+o(\Delta t)\}\times\{1-\mu_{k-1}\Delta t+o(\Delta t)\}$$

$$+P_{k+1}(t)\times\{1-\lambda_{k+1}\Delta t+o(\Delta t)\}\times\{\mu_{k+1}\Delta t+o(\Delta t)\}+o(\Delta t)$$

$$\Rightarrow P_k(t+\Delta t)-P_k(t) = -(\lambda_k+\mu_k)P_k(t)\times\Delta t+P_{k-1}(t)\lambda_{k-1}\times\Delta t+P_{k+1}(t)\mu_{k+1}\times\Delta t+o(\Delta t)$$

Dividing both sides byΔt and taking limit as $\Delta t \to 0$

$$\Rightarrow \frac{d}{dt}\{P_k(t)\} = -(\lambda_k+\mu_k)P_k(t)+\lambda_{k-1}P_{k-1}(t)+\mu_{k+1}P_{k+1}(t) \qquad (11)$$

Since, $\lim_{t\to\infty}\frac{o(\Delta t)}{\Delta t} = 0$

Here in state k data packet arrival is $(\lambda+k\gamma)$

i.e., $\lambda_k = \lambda + k\gamma$

Also in state k data packet departure is µ

i.e., $\mu_k = \mu$

Hence (11) reduces to

$$\frac{d}{dt}\{P_k(t)\} = -(\lambda + k\gamma + \mu)P_k(t) + \{\lambda + (k-1)\gamma\}P_{k-1}(t) + \mu P_{k+1}(t) \text{ , where } k = 1,2,\ldots,C \qquad (12)$$

Case 2:

For k =0, we have

$P_0(t+\Delta t)$ = prob {no data packet present in the system in time (t+Δt) }

= prob {no data packet present in time t } × prob { no data packet arrival in time Δt } + prob {one data packet present in time t} × prob {no data packet arrival In time Δt } × prob { one data packet departure in time Δt } .

$$= P_0(t) \times \{1 - \lambda_0 \Delta t + o(\Delta t)\} + P_1(t) \times \{1 - \lambda_1 \Delta t + o(\Delta t)\} \times \{\mu_1 \Delta t + o(\Delta t)\}$$

$$\Rightarrow P_0(t+\Delta t) - P_0(t) = -\lambda_0 P_0(t) + P_1(t)\mu_1 + o(\Delta t)$$

Dividing both sides by Δt and taking limit as Δt→0, we get

$$\Rightarrow \frac{d}{dt}\{P_0(t)\} = -\lambda P_0(t) + \mu P_1(t) \quad \text{, for } k=0 \qquad (13)$$

$$(\text{since, } \lambda_k = \lambda + k\gamma \text{ And } \mu_k = \mu)$$

Case 3:

For k =C+1, we have

$P_{C+1}(t+\Delta t)$ = prob { (C+1) no. of data packet present in the system in time (t+Δt)}

ISSN:2320-3714
Volume:2 Issue:3
June 2024
Impact Factor:10.2
Subject:Computer Science

= prob {C no. of data packet present in time t } × prob {1 data packet arrival in time Δt }× prob { no data packet departure in time Δt } + Prob { (C+1) no of data packets present in time t } × prob { no data packet departure in time Δt }

$$= P_C(t) \times \{\lambda_C \Delta t + o(\Delta t)\} \times \{1 - \mu_C \Delta t + o(\Delta t)\} + P_{C+1}(t) \times \{1 - \mu_{C+1}\Delta t + o(\Delta t)\}$$

$$\Rightarrow P_{C+1}(t+\Delta t) - P_{C+1}(t) = P_C(t)\lambda_C + o(\Delta t) - \mu_{C+1}P_{C+1}(t)$$

Dividing both sides by Δt and taking limit as Δt →0 we get

$$\frac{d}{dt}\{P_{C+1}(t)\} = P_C(t)\lambda_C - \mu_{C+1}P_{C+1}(t)$$

$$\Rightarrow \frac{d}{dt}\{P_{C+1}(t)\} = (\lambda + C\gamma)P_C(t) - \mu P_{C+1}(t) \qquad \text{(Since, } \lambda_C = \lambda + C\gamma \text{)} \qquad (14)$$

The instantaneous congestion rate $A_0(t)$ can be found by solving this system of differential equations as part of our analysis using Queuing Theory for heavy traffic in data transmission and communication networks.

$$A_0(t) = P_1(t) = \frac{\lambda}{\mu + \lambda}(1 - e^{-(\mu+\lambda)t})$$

To evaluate the system's steady operating state, the instantaneous congestion rate is not enough. Determining the stable congestion rate, which stays constant throughout time when the system functions in a stable condition, is therefore crucial. In the context of Heavy Traffic Analysis of Data Transmission and Communication Systems Using Queuing Theory, the stable congestion rate is defined as follows.

$$A_C = \lim_{t\to\infty} A_C(t)$$

Taking into account $P = \lim_{t\to\infty} P(t)$ as the queue's stable length distribution and C as the router's buffer, there are two methods to determine the stable congestion rate: first, we get the instantaneous congestion rate and then figure out its limit. Its definition states that it can be obtained by dispersing the queue's length. Second, we know from the Markov Process that a

ISSN:2320-3714
Volume:2 Issue:3
June 2024
Impact Factor:10.2
Subject:Computer Science

system of steady state equations can be used to distribute a queue's stable length. The differential difference equation system that results from equations (12), (13), and (14) is as follows.

$$\frac{d}{dt}\{P_k(t)\} = -(\lambda + k\gamma + \mu)P_k(t) + \{\lambda + (k-1)\gamma\}P_{k-1}(t) + \mu P_{k+1}(t) \quad \text{for k=1,2,3,..,C} \qquad (15)$$

$$\frac{d}{dt}\{P_0(t)\} = -\lambda P_0(t) + \mu P_1(t) \quad \text{for k =0} \qquad (16)$$

$$\frac{d}{dt}\{P_{C+1}(t)\} = (\lambda + C\gamma)P_C(t) - \mu P_{C+1}(t) \quad \text{for k=C+1} \qquad (17)$$

Certain Markov process properties state that

$P_i(t)$ = (i=0,1,2,…,C+1) is known to satisfy the differential equation above.

$$\text{Here } P(t) = [P_0(t), P_1(t),..., P_{C+1}(t)]$$

$$P(0) = [P_0(0), P_1(0),..., P_{C+1}(0)]$$

$$P_0(0) = 1, P_1(0) = 0, P_2(0) = 0,..., P_{C+1}(0) = 0$$

For steady state condition $\lim_{t\to\infty} \frac{d}{dt} P_k(t) = 0$ and $\lim_{t\to\infty} P_k(t) = P_k$

The balancing equations that follow are transformed into (15), (16), and 17 under steady state conditions.

$$0 = (\lambda + k\gamma + \mu)P_k + \{\lambda + (k-1)\gamma\}P_{k-1} + \mu P_{k+1} \quad \text{for k =1,2,3,… ,C} \qquad (18)$$

$$0 = -\lambda P_0 + \mu P_1, \text{ for k =0} \qquad (19)$$

$$0 = (\lambda + C\gamma)P_C - \mu P_{C+1} \quad \text{for k =C+1} \qquad (20)$$

The system of steady state equations mentioned above can be expressed in a matrix as

ISSN:2320-3714
Volume:2 Issue:3
June 2024
Impact Factor:10.2
Subject:Computer Science

$$PQ = 0$$

$$\sum_{i=0}^{C+1} P_i = 1$$

Where P = (P_0, P_1,…,P_{c+1})

And

$$Q = \begin{pmatrix} -\lambda & \lambda & 0 & 0 & \cdots & 0 & 0 \\ \mu & -(\lambda+\mu+v) & \lambda+v & 0 & \cdots & 0 & 0 \\ 0 & \mu & -(\lambda+\mu+2v) & \lambda+2v & \cdots & 0 & 0 \\ 0 & 0 & \mu & -(\lambda+\mu+3v) & \cdots & 0 & 0 \\ 0 & 0 & 0 & \mu & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -(\lambda+\mu+Cv) & \lambda+Cv \\ 0 & 0 & 0 & 0 & \cdots & \mu & -\mu \end{pmatrix}$$

For C= 0,

From (19) we get

$$\lambda P_0 = \mu P_1 \tag{21}$$

Also, $P_0 + P_1 = 1$ (22)

Solving (21) and (22) we get

$$P_1 = \frac{\lambda}{\lambda + \mu}$$

Hence

ISSN:2320-3714
Volume:2 Issue:3
June 2024
Impact Factor:10.2
Subject:Computer Science

$$A_0 = P_1 = \frac{\lambda}{\lambda + \mu} \qquad (23)$$

We determine that the steady-state congestion rate is

$$A_C = P_{C+1} = 1 - \frac{\mu}{(\lambda + \mu + C\gamma)A_{C-1} - \{\lambda + (C-1)\gamma\}(1 - A_{C-1})A_{C-2} + \mu} \text{ , for } C \geq 2$$

6. MODEL 2: THE TWO-SERVER QUEUING MODEL (M/M/2) FOR HEAVY TRAFFIC ANALYSIS (C+1/FCFS)

There are two servers or channels in this model, and they are arranged in parallel. With a mean rate of □ per unit time, the arrival distribution in this case is a Poisson distribution. With a mean rate of □ per unit time, the service time is exponential. Every server is the same, meaning that they all provide the same services at a mean fee of □ per unit of time. There are two ways to find the overall service rate. If the system has n different numbers of data packets.

Case-1

For $n < 2$

There won't be a queue. The aggregate service rate will be as a result of (2-n) servers remaining idle.

$$\mu_n = n\mu,\ 1 \leq n < 2$$

Case-2

For $n \geq 2$

After then, every server would be busy. Thus, the highest possible (n-2) (≤C+1) quantity of data packets in the queue. The total service charge will be

$$\mu_n = 2\mu,\ n \geq 2$$

Thus, when we combine Cases 1 and 2, we obtain

ISSN:2320-3714
Volume:2 Issue:3
June 2024
Impact Factor:10.2
Subject:Computer Science

$$\lambda_n = \lambda, \ \ for\ all\ n \geq 0$$

$$\mu_n = n\mu, 1 \leq n < 2$$

$$\mu_n = 2\mu, n \geq 2$$

$$\mu_0 = 0, n = 0$$

$$\mu_1 = \mu, n = 1$$

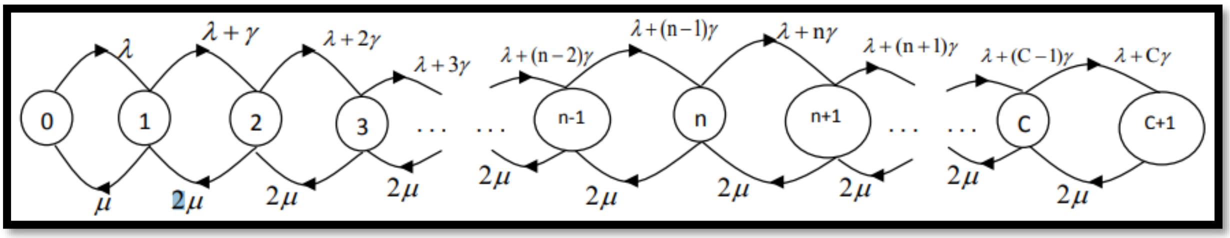

Figure 3: Diagram of the state transition rate

The formulas for the steady state are,

$$\lambda P_0 = \mu P_1, \ \ For\ n = 0 \tag{24}$$

$$(\lambda + \gamma + \mu)P_1 = \lambda P_0 + 2\mu P_2, \ \ For\ n = 1 \tag{25}$$

$$\{\lambda + (n-1)\gamma\}P_{n-1} + 2\mu P_{n+1} = (\lambda + n\gamma)P_n + 2\mu P_n, \ \ for\ 2 \leq n \leq C \tag{26}$$

$$(\lambda + C\gamma)P_C = 2\mu P_{C+1}, \quad For\ n = C + 1 \tag{27}$$

The system of steady state balancing equations mentioned above can be expressed as a matrix as

$$PQ = 0$$

And $\sum_{i=0}^{C+1} P_i = 1$

Where P = $(P_0, P_1, \ldots, P_{c+1})$

And

AIRO JOURNALS | Peer Reviewed Multidisciplinary International
ISSN:2320-3714
Volume:2 Issue:3
June 2024
Impact Factor:10.2
Subject: Computer Science

$$Q = \begin{pmatrix} -\lambda & \lambda & 0 & 0 & \dots & 0 & 0 \\ \mu & -(\lambda+\mu+\nu) & \lambda+\nu & 0 & \dots & 0 & 0 \\ 0 & 2\mu & -(\lambda+2\mu+2\nu) & \lambda+\nu & \dots & 0 & 0 \\ 0 & 0 & 2\mu & -(\lambda+2\mu+3\nu) & \dots & 0 & 0 \\ 0 & 0 & 0 & 2\mu & \dots & 0 & 0 \\ . & . & . & . & \dots & . & . \\ . & . & . & . & \dots & . & . \\ 0 & 0 & 0 & 0 & \dots & -(\lambda+2\mu+C\nu) & \lambda+C\nu \\ 0 & 0 & 0 & 0 & \dots & 2\mu & -2\mu \end{pmatrix}$$

For C=0 we have

$$\lambda P_0 = \mu P_1 \qquad (28)$$

$$\text{Also, } P_1 + P_0 = 1 \qquad (29)$$

From (29) we get $P_0 = 1 - P_1$

Then (28) becomes

$$\lambda(1 - P_1) = \mu P_1$$

$$\Rightarrow P_1 = \frac{\lambda}{\lambda + \mu}$$

Hence

$$A_0 = P_1 = \frac{\lambda}{\lambda + \mu}$$

We determine that the steady-state congestion rate is

$$A_C = P_{C+1} = 1 - \frac{2\mu}{(\lambda + 2\mu + C\gamma)A_{C-1} - \{\mu + (C-1)\gamma\}(1 - A_{C-1})A_{C-2} + 2\mu} \ for\ C \geq 2$$

7. CONCLUSION AND RECOMMENDATIONS

Using Queuing Theory, this research programme explores the heavy traffic analysis of communication and data transmission networks. In order to efficiently estimate and handle large traffic, we are concentrating on building and analysing queuing models. Specifically, we use two queuing models to anticipate and stabilise congestion rates in heavy traffic: (M/M/1): ((C+1)/FCFS and (M/M/2): ((C+1)/FCFS. These models are essential for predicting future congestion points and comprehending the behaviour of heavy traffic under various circumstances. The utilisation of these models from Queuing Theory offers a simple and effective way to calculate and track heavy traffic in communication networks. We can efficiently monitor and control network performance by gaining a more comprehensive and accurate picture of traffic flow through the analysis of these models. This is especially crucial for preserving network dependability and efficiency across a variety of traffic scenarios, such as ideal, normal, and high-overhead ones. In the end, our research emphasises how important heavy traffic rate analysis is to the field of communication systems and data transmission. We can make sure that heavy traffic is watched over and handled in a way that improves the overall performance and stability of communication networks by utilising Queuing Theory models. This study emphasises how crucial accurate traffic management techniques are to the upkeep of reliable and effective network communication systems.

Beyond (M/M/1) and (M/M/2) models, future research should examine complicated systems such as (M/G/1) and (G/G/1), investigate dynamic and intermittent traffic patterns, and assess the effects of newly developing technologies like 5G and IoT. Our understanding and management of heavy traffic in data transmission and communication systems will be further improved by integrating machine learning with queuing theory, examining scalability in large networks, examining QoS and QoE impacts, optimising resource allocation strategies, carrying out extensive simulations and real-world testing, researching the effects of security measures on traffic patterns, and investigating policy and regulatory implications.

REFERENCES

[1] Anselmi, J., & Casale, G. (2013). Heavy-traffic revenue maximization in parallel multiclass queues. Performance Evaluation, 70(10), 806-821.

ISSN:2320-3714
Volume:2 Issue:3
June 2024
Impact Factor:10.2
Subject:Computer Science

[2] Miyawaki, H., Masuyama, H., & Takahashi, Y. (2013). Heavy-traffic asymptotic formulas for the multiclass FIFO M X/G/1 queue with conjectures for a general queueing model. arXiv preprint arXiv:1312.6970.

[3] Ata, B., & Barjesteh, N. (2023). An approximate analysis of dynamic pricing, outsourcing, and scheduling policies for a multiclass make-to-stock queue in the heavy traffic regime. Operations Research, 71(1), 341-357.

[4] Ata, B., & Peng, X. (2018). An equilibrium analysis of a multiclass queue with endogenous abandonments in heavy traffic. Operations Research, 66(1), 163-183.

[5] Cohen, A. (2019). Asymptotic analysis of a multiclass queueing control problem under heavy traffic with model uncertainty. Stochastic Systems, 9(4), 359-391.

[6] Gurvich, I. (2014). Validity of heavy-traffic steady-state approximations in multiclass queueing networks: The case of queue-ratio disciplines. Mathematics of Operations Research, 39(1), 121-162.

[7] Sani, S., & Daman, O. A. (2014). Mathematical modeling in heavy traffic queuing systems. American Journal of Operations Research, 4(06), 340.

[8] Izagirre, A., Verloop, I. M., & Ayesta, U. (2015). Heavy-traffic analysis of a non-preemptive multi-class queue with relative priorities. Probability in the Engineering and Informational Sciences, 29(2), 153-180.

[9] Atar, R., & Cohen, A. (2016). A differential game for a multiclass queueing model in the moderate-deviation heavy-traffic regime. Mathematics of Operations Research, 41(4), 1354-1380.

[10] Atar, R., & Cohen, A. (2017). Asymptotically optimal control for a multiclass queueing model in the moderate deviation heavy traffic regime.

[11] Atar, R., & Shifrin, M. (2015). An asymptotic optimality result for the multiclass queue with finite buffers in heavy traffic. Stochastic Systems, 4(2), 556-603.

[12] Budhiraja, A., Ghosh, A., & Liu, X. (2012). Dynamic scheduling for Markov modulated single-server multiclass queueing systems in heavy traffic. arXiv preprint arXiv:1211.6831.

[13] Budhiraja, A., Ghosh, A., & Liu, X. (2014). Scheduling control for Markov-modulated single-server multiclass queueing systems in heavy traffic. Queueing Systems, 78(1), 57-97.

ISSN:2320-3714
Volume:2 Issue:3
June 2024
Impact Factor:10.2
Subject:Computer Science

[14] Dai, J. G., & Huo, D. (2024). Asymptotic Product-form Steady-state for Multiclass Queueing Networks with SBP Service Policies in Multi-scale Heavy Traffic. arXiv preprint arXiv:2403.04090.

[15] Gamarnik, D., & Stolyar, A. L. (2012). Multiclass multiserver queueing system in the Halfin–Whitt heavy traffic regime: Asymptotics of the stationary distribution. Queueing Systems, 71(1), 25-51.

[16] Hillas, L. A., Caldentey, R., & Gupta, V. (2024). Heavy traffic analysis of multi-class bipartite queueing systems under FCFS. Queueing Systems, 1-46.

[17] Hurtado-Lange, D., & Maguluri, S. T. (2022). A load balancing system in the many-server heavy-traffic asymptotics. Queueing Systems, 101(3), 353-391.

[18] Katsuda, T. (2012). Stationary distribution convergence for a multiclass single-server queue in heavy traffic. Scientiae Mathematicae Japonicae, 75(3), 317-334.

[19] Liu, Z., Chu, Y., & Wu, J. (2016). Heavy-traffic asymptotics of a priority polling system with threshold service policy. Computers & Operations Research, 65, 19-28.

[20] Zhou, X., & Shroff, N. (2020). A Note on Stein's Method for Heavy-Traffic Analysis. arXiv preprint arXiv:2003.06454.

Author's Declaration

I as an author of the above research paper/article, here by, declare that the content of this paper is prepared by me and if any person having copyright issue or patent or anything otherwise related to the content, I shall always be legally responsible for any issue. For the reason of invisibility of my research paper on the website /amendments /updates, I have resubmitted my paper for publication on the same date. If any data or information given by me is not correct, I shall always be legally responsible. With my whole responsibility legally and formally have intimated the publisher (Publisher) that my paper has been checked by my guide (if any) or expert to make it sure that paper is technically right and there is no unaccepted plagiarism and hentriacontane is genuinely mine. If any issue arises related to Plagiarism/ Guide Name/ Educational Qualification /Designation /Address of my university/ college/institution/ Structure or Formatting/ Resubmission /Submission /Copyright /Patent /Submission for any higher degree or Job/Primary Data/Secondary Data Issues. I will be solely/entirely responsible for any legal issues. I have been informed that the most of the data from the website is invisible or shuffled or vanished from the database due to some technical fault or hacking and therefore the process of resubmission is there for the scholars/students who finds trouble in getting their paper on the website. At the time of resubmission of my paper I take all the legal and formal

ISSN:2320-3714
Volume:2 Issue:3
June 2024
Impact Factor:10.2
Subject:Computer Science

responsibilities, If I hide or do not submit the copy of my original documents (Andhra/Driving License/Any Identity Proof and Photo) in spite of demand from the publisher then my paper maybe rejected or removed from the website anytime and may not be consider for verification. I accept the fact that as the content of this paper and the resubmission legal responsibilities and reasons are only mine then the Publisher (Airo International Journal/Airo National Research Journal) is never responsible. I also declare that if publisher finds Any complication or error or anything hidden or implemented otherwise, my paper maybe removed from the website or the watermark of remark/actuality maybe mentioned on my paper. Even if anything is found illegal publisher may also take legal action against me

Krati Garg

*****