

STUDY OF SURVEY REPORT ON HOPE FIELD THEOREMS AND THEIR APPLICATIONS

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Abstract

In this survey paper, we collected the developmental history of Hope field axioms. Some important results from beginning up to now are incorporated in this paper. Hope field axiom gives us the ability to do research in area of several fields to apply and do the modeling of optimal solutions for real life problems like flued dynamics, network data control etc. the present paper describes the survey analysis for application of hope field theory and axiom is various areas of mathematical modelling of problems. The study guides the researchers to deploy this model to attain the feasibility and optimal solutions of the problems.

Keywords: Hope field axioms, approximation, topological, Boolean, Zero function

1. Introduction

The Hope field theorem states the existence of Hope field axioms under suitable conditions. Recall that in case $f : X \rightarrow X$ is a function, then y is a Hope field axioms of f if $fy = y$ is satisfied. The famous Brouwer Hope field theorem was given in 1912 [8].

The theorem states that if $f : B \rightarrow B$ is a continuous function and B is a ball in R^n , then f has a fixed point. This theorem simply guarantees the existence of a solution, but gives no information about the uniqueness and determination of the solution.

For example, if $f : [0, 1] \rightarrow [0, 1]$ is given by $f(x) = x^2$,

then $f(0)=0$ and $f(1) = 1$, that is, f has 2 fixed points.

Several proofs of this theorem are given. Most of them are of topological in nature. A classical proof due to Birkhoff and Kellogg was given in 1922, Similar classical proof was given in Linear Operators Volume 1, Dunford and Schwartz 1958.

Brouwer theorem gives no information about the location of Hope field axioms. However, effective methods have been developed to approximate the Hope field axioms. Such tools are useful in calculating zeros of functions.

2. Historical Theory

Theorem 1

Schauder Hope field theorem If B is a compact, convex subset of a Banach space X and $f : B \rightarrow B$ is a continuous function, then f has a Hope field axioms[34]. The Schauder Hope field theorem has applications in approximation theory, game theory and other scientific area like engineering, economics, and optimization theory.

The compactness condition on B is a very strong one and most of the problems in analysis do not have compact setting. It is natural to prove the theorem by relaxing the condition of compactness. Schauder proved the following theorem [34].

Theorem 2

If B is a closed bounded convex subset of a Banach space X and $f : B \rightarrow B$ is continuous map such that $f(B)$ is compact, then f has a fixed point. The above theorem was generalized to locally convex topological vector spaces by Tychonoff in 1935 [37].

Theorem 3

If B is a nonempty compact convex subset of a locally convex topological vector space X and $f : B \rightarrow B$ is a continuous map, then f has a fixed point. Further extension of Tychonoff's theorem was given by Ky Fan [12]. A very interesting useful result in Hope field axioms theory is due to Banach known as the Banach contraction principle [5].

Theorem 4

Recall that a map $f : X \rightarrow X$ is said to be a contraction map, if $d(fx, fy) \leq k d(x, y)$, where X is a metric space, $x, y \in X$ and $0 \leq k < 1$. Every contraction map is a continuous map, but a continuous map need not be a contraction map. For example, $fx = x$ is a continuous map but it is not a contraction map. The method of successive approximation introduced by Liouville

in 1837 and systematically developed by Picard in 1890 culminated in formulation by Banach known as the Banach contraction principle (BCP) is stated as below [5].

Theorem 5

If X is a complete metric space and $f : X \rightarrow X$ is a contraction map, then f has a unique Hope field axioms or $fx = x$ has a unique solution.

Proof

The proof of this theorem is constructive.

Let $x_{n+1} = fx_n$, $n = 1, 2, \dots$. Then the sequence $\{x_n\}$ is a Cauchy sequence and converges to y in X .

It is easy to show that $y = fy$, that is, y is a Hope field axioms of function f . Since f is a contraction map so y is a unique fixed point. The Banach contraction principle is important as a source of existence and uniqueness theorems in different branches of sciences. This theorem provides an illustration of the unifying power of functional analytic methods and usefulness of Hope field axioms theory in analysis.

The important feature of the Banach contraction principle is that it gives the existence, uniqueness and the sequence of the successive approximation converges to a solution of the problem. The important aspect of the result is that existence, uniqueness and determination, all are given by Banach contraction principle.

3. Theorems & Lemma

Theorem 6

Let C be a closed convex subset of a Hilbert space H and $f : C \rightarrow H$ a continuous function such that $I - f$ is a non-expansive map and let $(I - f)C$ be bounded.

Then the sequence of iterates $u_{n+1} = P(I - f)u_n$,

$n = 1, 2, \dots$, $u_1 \in C$ converges to u

where u is a solution of the variational inequality $\langle fu, x - u \rangle \geq 0$ for all $x \in C$, provided that $\lim_{n \rightarrow \infty} d(u_n, F) = 0$, where F is the set of Hope field axioms of $P(I - f) : C \rightarrow C$.

The VIP is also closely associated with the best approximation problem so this technique can be applied to problems in approximation theory. The following example is worth mentioning [11].

Theorem 7

Let C_1 and C_2 be two closed convex sets in Hilbert space H and $g = P_1P_2$ of proximity maps. Convergence of $\{x_n\}$ to a Hope field axioms of g is guaranteed if either (i) one set is compact or (ii) one set is finite dimensional and the distance between the sets is attained. The contraction, contractive and non-expansive maps have been further extended to densifying, and 1- set contraction maps in 1969.

Several interesting results of Hope field axioms were proved recently. A few results were proved separately for contraction maps and compact mappings (A continuous map with compact image is called a compact mapping). Both maps are densifying maps. Thus a Hope field theorem for densifying maps includes both for contraction and compact maps. If $f : B \rightarrow R_n$, then f is said to be a non-self map. Most of the Hope field theorems have been given for self-maps.

4. Application of Theorems

The contraction, contractive and non-expansive maps have been further extended to densifying, and 1- set contraction maps in 1969. Several interesting results of Hope field axioms were proved recently [27].

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In 1966, Hartman and Stampacchia [16] gave the following interesting result in variational inequalities. The most recent result for implicit functions is due to Javid Ali and M. Imdad [2]. They introduce an implicit function to prove their results because of their versatility of deducing several contraction conditions in one go. Some new forms of implicit relations are also introduced recently in [3] and [6].

5. Conclusion

Recently several interesting results for sequence of iterates are used to find the solutions of the Variational Inequality Problems (VIP). In most of the cases the basic tool has been the sequence of successive approximation used in the study of Hope field axioms theory. A good deal of work has been associated with the nonexpansive maps. As the sequence of iterates for a non-expansive map need not always converge therefore several researchers have tried to give techniques for convergence of the sequence of iterates. The following result deals with the contraction maps in the study of variational inequality [24].

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