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AN INTRODUCTION TO INNOVATIVE SURVIVAL DISTRIBUTIONS BY ANALYZING THEIR UTILITY WITH TWO REAL-WORLD DATASET APPLICATIONS

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Abstract

Within the constraints of this study, we offer a Modi family of continuous probability distributions that is exhaustive in its scope. The persons who are afflicted with sickness and the lengths of time they are able to survive are the targets of this family's application. The application focuses on the folks who are able to survive. A density function that has three parameters and a hazard rate function that has an inverted J-shape are two of the characteristics of the distribution that have been described in depth with regard to the distribution. Each of these density functions will be discussed in more detail below. Through the utilization of its mathematical and statistical capabilities, which we applied in order to carry out the inquiry, we were able to analyze the attributes of the distribution that was recommended. Additionally, the probability density function *of order statistics may be derived for this distribution. This is something that can be done. Certainly, this is something that is attainable. The maximum likelihood estimate is the approach that we employ in order to carry out the conventional way of estimating parameters. This method is known as the maximum likelihood estimate. It is possible for us to do things in this way. For the purpose of proving that our distribution provides a better match than other well-known distributions, we applied it to two real datasets and demonstrated that it delivered. It is via the utilization of this distribution that this shall be proved.*

Keywords: *Innovative Survival Distributions, Applications, Modi family, Two Real-World Dataset.*

1. Introduction

A mathematical model of a real-world scenario aids in our understanding and explanation of it. Additionally, it could enable us to reproduce the issue at a bigger scale or one that only highlights the key components of the phenomenon. These phenomena may be captured by probability distribution models, which can be taught or abstracted from gathered data on actual events. Many fields use lifetime models with both monotone and non-monotone failure rate aspects, such as engineering, life sciences, finance and insurance, environmental sciences, medical sciences, biological studies, demography, actuaries, and economics. In order to better study and apply lifetime data sets in many sectors, distribution theory has concentrated on developing new models and resolving research challenges in the present day.

We have suggested the Modi family of distributions in recognition of the need for more adaptable lifetime distributions. We have specifically covered the Modi exponential distribution, which has three parameters and may be applied to fit and analyze data in several domains. The following is how the paper is organized: In Section, we define the exponential distribution, and in Modi generator. The Modi exponential distribution's cumulative distribution function (PDF) and probability density function (PDF) are presented in Section. The expression for the derived distribution's survival function and hazard rate function may be found in Section. The suggested distribution's mode and median are provided in Section. Sections provide the formulas for calculating the Modi exponential distribution's moments and probability weighted moments, respectively. In Section and Section, respectively, the Renyi entropy and distribution of the order statistics for the new distribution are covered. In Section, the parameters of the resulting distribution are estimated using the maximum likelihood estimation approach. We demonstrate the Modi exponential distribution's application on two real datasets in Section and contrast it with a few other well-known distributions. To determine the outcome of the derivations, we will require the following lemmas:

When |x| < 1 and α is a positive real non-integer, the binomial series expansion yields:

$$(1-x)^{a-1} = \sum_{j=0}^{\infty} (-1)^j \binom{a-1}{j} x^j \qquad (1)$$

For $a < 1, q > -1, Re(p) > 0$,

$$\int_0^{\infty} \frac{x^q e^{-px}}{(1-ae^{-px})^2} dx = \frac{\Gamma(q+1)}{ap^{(q+1)}} \sum_{k=1}^{\infty} \frac{a^k}{k^q} \qquad (2)$$

In some industries, such as finance, architecture, and healthcare, it is essential to comprehend how long an event of interest will take place. This is where the importance of survival analysis comes in. The last few years have seen an explosion in the creation of inventive survival vehicles, offering substitutes for managing sporadic information displays. As an introduction to them, this research looks at the theoretical underpinnings and real-world applications of these unique conveyances. We want to demonstrate these dispersions' versatility and usefulness for detecting complex survival oddities through an investigation of two real-world dataset applications. In addition to deepening our understanding of these methods, this work attempts to provide light on the potential effects of survival analysis approaches on dynamic cycles in many fields.

2. Literature Review

Alzaatreh, Lee, and Famoye (2013) suggested a creative process for generating sets of reliable circulations. By providing a component to create distinct circulation families, their approach offers flexibility in quantifiable showing and hence improves the relevance of factual analysis across various datasets. Expanding the pool of available vehicles gives scientists more flexibility when presenting intricate information structures and researching theoretical hypotheses.

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Barreto-Souza, Cordeiro, and Simas (2011) explored properties of the beta Fréchet appropriation, adding to the comprehension of its measurable attributes and likely applications. Their discoveries shed light on the circulation's way of behaving and reasonableness for displaying different peculiarities in various fields. Seeing such properties is critical for really using the circulation in viable settings, going from risk analysis to unwavering quality designing.

Cooray and Ananda (2005) looked at using a composite lognormal-Pareto model to present actuarial data. The complex examples found in actuarial information sometimes call for the use of contemporary demonstration techniques to ensure accurate representation and analysis. The authors of this composite model provide a framework for understanding the intricacies of actuarial data, allowing for the use of more robust risk assessment and cutting-edge methods in the fields of money and protection.

Cordeiro, Afify, Yousof, et al. (2017) introduced the exponentiated Weibull-H group of dispersions, presenting theoretical constructions as well as real-world applications. This collection of dispersions provides a flexible framework for presenting survival data, which may find use in reliability assessment, design, and healthcare.

Gradshteyn and Ryzhik's (2007) The foundational book Table of Integrals, Series, and Products is still a vital tool for mathematical study and application. Researchers, engineers, and students may solve hard mathematical issues in a variety of fields with the help of this comprehensive reference, which gathers a vast collection of mathematical formulae, integrals, series, and products. This article is a necessary tool for anybody working in theoretical or practical mathematics because of its careful arrangement and thorough treatment of mathematical identities and functions.

Ferreira and Steel (2006) enhanced the text by presenting a useful illustration of univariate slanted disseminations. By means of their meticulous analysis, they revealed a brilliant solution to the problem of showing skewed data, providing a useful framework that enhances our ability to comprehend unequal delivery. Their work expands the body of quantifiable systems and provides experts with powerful tools for analyzing data that has inherent skewness, which advances the study of factual hypothesis and its application.

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3. Modi Family

A novel family of Modi probability distributions is presented herein to represent lifespan or survival data. The Modi generator's CDF F (x) and PDF f (x) are provided, respectively, by:

$$F(x) = \frac{(1+\alpha^{\beta})S(x)}{\alpha^{\beta}+S(x)} \tag{3}$$

$$F(x) = \frac{(1+\alpha^{\beta})\left(\alpha^{\beta}s(x)\right)}{\{\alpha^{\beta}+s(x)\}^{2}} \tag{4}$$

3.1.CDF and PDF of Modi Exponential Distribution

We suggest a new Modi exponential distribution for the Modi generator utilizing the PDF and CDF described in Eq, respectively. Consequently, one definition of the Modi exponential distribution's CDF is:

$$F(x) = \frac{\left(1+\alpha^{\beta}\right)\left(1-e^{-px}\right)}{\alpha^{\beta}+1-e^{-px}} \tag{5}$$

and its corresponding PDF is given by:

$$f(x) = \frac{p\alpha^{\beta}e^{-px}}{\left(1-\frac{e^{-px}}{1+\alpha^{\beta}}\right)^{2}\left(1+\alpha^{\beta}\right)} \tag{6}$$

for all $x > 0, where\ \alpha > 0, \beta > 0, p > 0$

Figures 1 and 2 display graphs of the Modi exponential distribution's density function and distribution function for various combinations of values for its parameters, p, β, and α.

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Table 1: A Modi exponential distribution's distribution function graph for various values of its parameters, p, β, and α

	0	0.2	0.4	0.6	0.8	1
α =0.3, β=5, p=0.5	0.5	1	1	1	1	1
α =0.3, β=0.5, p=5	0	0.8	0.9	1	1	1
α =0.3. β=5. p=5	0.2	1	1	1	1	1
α =1, β=0.5. p=0.5	0	0.1	0.3	0.4	0.5	0.6

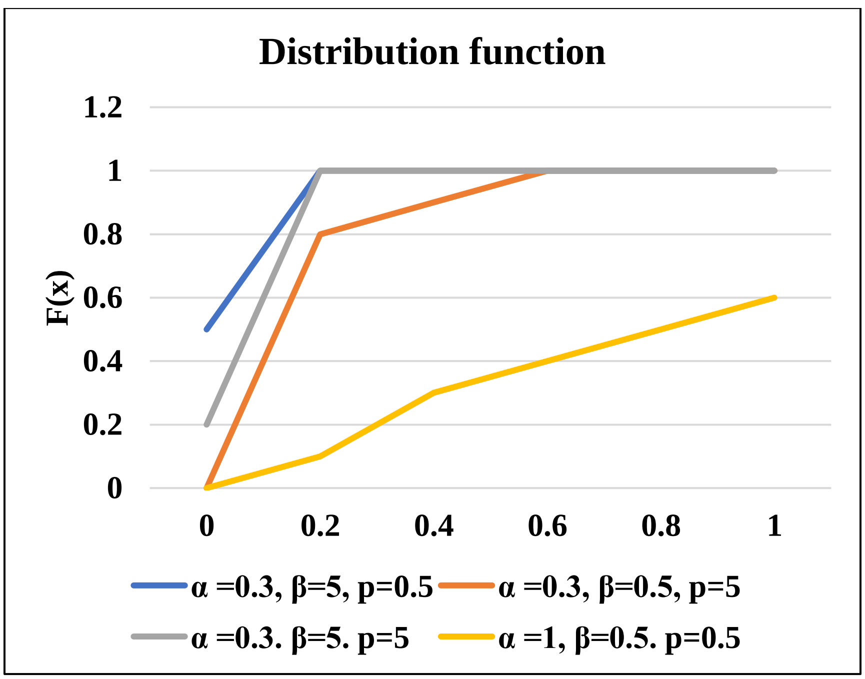

Figure 1: An illustration of the Modi exponential distribution's distribution function for various combinations of parameter values (p, β, and α)

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Table 2: A Modi exponential distribution's density function graph for various values of its parameters, p, β, and α.

	0	0.2	0.4	0.6	0.8	1
α =1, β=0, p=5	9	2	1	0	0	0
α =0.3, β=0.5, p=5	11	2	1	0	0	0
α =0.3, β=5, p=0.5	8	0	0	0	0	0
α =0.3, β=5, p=0.5	14	0	0	0	0	0

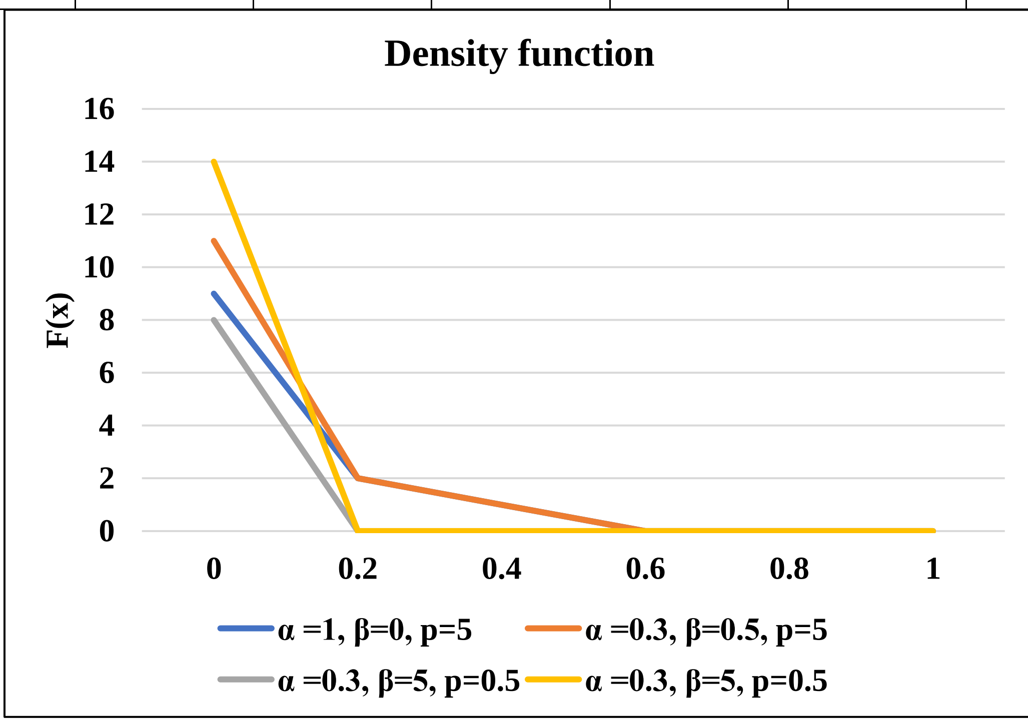

Figure 2: A Modi exponential distribution's density function graph for various values of its parameters, p, β, and α

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4. Application To Real Life Data

This section applies the suggested Modi exponential distribution to two real-world datasets. We note that it is flexible over several popular current distributions. R program was used to acquire the analytic findings for the current study. In order to assess the fit of the distributions under consideration, we have additionally computed the p-value and Akaike Information Criteria (AIC). In the meanwhile, it is thought that the distribution with the lowest AIC or the highest log-likelihood value is the best. The following is the PDF of the distributions made:

- **Burr-XII distribution**

$$f(x) = \frac{ck}{\alpha}\left(\frac{x}{\alpha}\right)^{c-1}\left(1+\left(\frac{x}{\alpha}\right)^{c}\right)^{-(k+1)} \quad (7)$$

- **Log-logistic distribution:**

$$f(x) = \frac{\beta\left(\frac{x}{\alpha}\right)^{\beta-1}}{\alpha\left(1+\left(\frac{x}{\alpha}\right)^{\beta}\right)^{2}}$$

(8)

- **Dagum distribution:**

$$f(x) = \beta\lambda\delta^{-(1+\delta)}\left(1+\lambda x^{-\delta}\right)^{1(1+\beta)} \quad (9)$$

- **Modi exponential distribution:**

$$f(x) = \frac{p a^{\beta} e^{-px}}{(1+a^{\beta})\left(1-\frac{e^{-px}}{1+a^{\beta}}\right)^{2}} \quad (10)$$

- **Weibull distribution:**

$$f(x) = \frac{\alpha}{\beta^{\alpha}} x^{a-1} rxp\left(\left(\frac{x}{\beta}\right)^{\alpha}\right)$$

(11)

Data Set 1:

The survival durations (in weeks) of 33 individuals with acute myelogenous leukemia are included in this data set. These data have been examined. 65, 156, 100, 134, 16, 108, 121, 4, 39, 143, 56, 26, 22, 1, 1, 5, 65, 56, 65, 17, 7, 16, 22, 3, 4, 2, 3, 8, 4, 3, 30, 4, and 43 are the given statistics. We will contrast the suggested Modi exponential distribution with a few well-known distributions for this set of data. Assume for the moment that α=1% LOS,

H0: The data follow the Modi exponential distribution.

H1: The data do not follow the Modi exponential distribution.

Data Set 2:

This dataset includes 50 device failure times that Aarset examined. The information is as follows: 0.1, 0.2, 1, 1, 1, 1, 2, 3, 6, 7, 11, 12, 18, 18, 18, 18, 21, 32, 36, 40, 45, 46, 47, 50, 55, 60, 63, 67, 67, 67, 72, 75, 79, 82, 82, 83, 84, 84, 85, 85, 85, 86, and 86. For Data Sets 1 and 2, the ML estimates of the Modi exponential distribution parameters and the AIC value are provided in.

The Modi exponential distribution fits both data sets better than the Weibull, Dagum, Log-Logistic, and Burr-XII distributions since it has the lowest AIC value and the highest log-likelihood value among other well-known distributions. AIC = −2 log eL + 2k is one way to compute the AIC, where logeL is the log-likelihood function that is determined using maximum likelihood estimations, and k is the number of parameters. Since the p-value is greater than α, the null hypothesis cannot be rejected, leading us to believe that the data has a Modi exponential distribution.

5. Conclusion

The innovative family of distributions we have introduced in this article is called the Modi family. The formulas for the exponential distribution's probability density function (PDF) and cumulative distribution function (PDF) are computed. The researchers also looked at the formed distribution's statistical and mathematical characteristics. By examining the graphs made for the probability density function (PDF) of the estimated distribution for different parameter combinations, we can observe that it has a reverse "J" shaped density function. Furthermore, the survival function and hazard rate function graphs for the new distribution are displayed. The probability weighted moments, mode, and median of the data can all be found. For your review, the formula for its rth moment of derived distribution is provided below. We have also derived the formulas for the probability density function (PDF) of its r th order statistics and the Renyi entropy. The method used to estimate parameters is called Maximum Likelihood Estimation (MLE). Furthermore, the distribution is fitted to two real data sets and contrasted with other well-known distributions. The results show that compared to several other well-known distributions, the Modi Exponential distribution provides a more adequate fit.

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