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SIGNIFICANCE OF ITERATIVE METHODS OF SOLUTIONS AND ITS EQUILIBRIUM PROBLEMS

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ABSTRACT

In the most recent decades, the theory of optimization methods has been created exceptionally significantly for its extensive variety of utilizations in many research zones. It is realized that the optimization issues and numerous others imperative mathematical issues are firmly identified with variational inequality issues and equilibrium issues. Consequently the hypothesis of variational inequality and equilibrium issues are likewise been produced and are extremely intriguing themes of current investigations. Variational inequality and equilibrium issues bring together various well-known issues of connected and pure mathematics. Accordingly, rather than considering assortments sorts of issues differently, in some cases it is helpful to examine a solitary issue like equilibrium issue or variational inequality issue, which covers a huge scope of issues.From that point forward, from theoretical and practical perspective, the variational imbalance issues have an extraordinary significance.Various issues emerging in operation research, economics, diversion theory, mathematical physics and different ranges can likewise be consistently displayed as a variational imbalance issue over product of sets. The first variational disparity issue defined on the product of sets into an arrangement of variational imbalances, which is anything but difficult to settle, to build up some arrangement strategies for variational disparity issue over product of sets.

KEYWORDS: Iterative Methods, Solutions, Certain Classes, Equilibrium Problems, equilibrium issues

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INTRODUCTION

Recently, the Recently of variational inequality and equilibrium issues have emerged as an interesting branch of applicable arithmetic and turn into a rich wellspring of inspiration and inspiration for the investigation of an extensive number of issues arising in economics, optimization, and operations examine in a general and unified way Variational inequality hypothesis was started independently in the mid 1960's to examine the issues in the flexibility and potential hypothesis, respectively. The primary general hypothesis for the existence and uniqueness of solution of variational inequality was demonstrated by Lions and Stampacchia in 1967. From that point forward, from theoretical and practical perspective, the variational inequality problems have an incredible significance. It is notable that the variational inequality hypothesis has played a basic and vital part in the investigation of an extensive variety of problems arising in material science, mechanics, versatility, optimization, control hypothesis, administration science, operations explore, economics, transportation and different branches of mathematical and engineering sciences, the terminology of equilibrium issue was embraced by Blum and Oettli .They examined existence hypotheses and variational standard for equilibrium issues. From that point forward different generalizations of equilibrium issues considered by Blum and Oettli have been presented and examined by many creators. It is realized that the equilibrium issue has an awesome effect and impact in the development of a few points of science and engineering. It worked out that the speculations of many surely understood issues could be fitted into the hypothesis of equilibrium issues. It has been demonstrated that the hypothesis of equilibrium issue gives a natural, novel and unified system for a few issues arising in nonlinear examination, optimization, economics, fund, diversion hypothesis, material science and engineering.

Some tools of nonlinear functional analysis

Throughout the thesis unless otherwise expressed, H means a genuine Hilbert space; $H*$ signifies the topological double of H, we signify the standard and internal result of H by k • k, and h., .I respectively. Give C a chance to be a nonempty, shut and convex subset of H. Give $\{x_n\}$ a chance to be any sequence in H, at that point $x_n \to x$ (respectively, $x_n \rightharpoonup x$) will mean solid (respectively

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feeble) convergence of the sequence $\{x_n\}$ $\mathbb{R}$ indicates the arrangement of every single genuine number.

Definition1.. [154 $Let\ T : C \to C$ be a mapping. A point x_0 is called a fixed point of T , $T,\ if\ Tx_0 = x_0,\ for\ all\ x_0 \in C,\ i.e.,$ a point which remains invariant under the transformation The fixed point problem (in short, FPP) for the mapping *T* is to find $x \in C$ such that

$$x = T x. \quad (1)$$

We denote Fix (*T*), the set of solutions of FPP (1).

Definition1..*Let X* be nor med linear space. A mapping $T: X \to X$ is said to be:

(1) Continuous at a arbitrary point $x_0 \in X$ if for each $\epsilon > 0$ there is real number $\delta > 0$ such that

$$x \in X, \|x - x_0\| < \delta \Rightarrow \|T(x) - T(x_0)\| \leq \epsilon, \quad \forall\, x_0 \in X;$$

(2) Lipschitz continuous if there exists a real constant $k > 0$ such that

$$\|T(x) - T(y)\| \leq k\|x - y\| \quad \forall\, x, y \in X;$$

(3) Contraction if it is Lipchitz continuous with $k \in (0, 1)$;

(4) Non-expansive if it is Lipschitz continuous with $k = 1$.

Theorem 1. (Banach Contraction Theorem) *Let X* be a complete nor-med linear space and $T: X \to X$ be a contraction mapping on *X*. Then FPP (1) has a unique solution in *X*.

Remark1. It is well known that every non-expansive operator $T: H \to H$ satisfies, for all $(x, y) \in H \times H,$ the inequality

$$\langle (x - T(x)) - (y - T(y)), T(y) - T(x) \rangle \leq (1/2)\|(T(x) - x) - (T(y) - y)\|^2 \quad (2)$$

And therefore, we get, for all $(x, y) \in H \times \mathrm{Fix}(T),$

, $\langle x - T(x), y - T(x)\rangle \le (1/2)\|T(x) - x\|^2,$ (3)

Definition 2. *Let T: H → H* be a nonlinear mapping. Then T is called:

(i) Monotone, if

$\langle Tx - Ty, x - y\rangle \ge 0, \quad \forall x, y \in C;$

(ii) α-strongly monotone, if there exists a constant α > 0 such that

$\langle Tx - Ty, x - y\rangle \ \ge \alpha\|x - y\|^2, \quad \forall x, y \in H;$

(iii) β-inverse strongly monotone, if there exists a constant β > 0 such that

$\langle Tx - Ty, x - y\rangle \ \ge \beta\|Tx - Ty\|^2, \quad \forall x, y \in H;$

(iv) Firmly non-expansive, *if it is β*-inverse strongly monotone with $\beta = 1$.

It is easy to observe that every β-inverse strongly monotone mapping T is monotone and $\beta^{\frac{1}{\beta}}$-Lipschitz continuous.

Definition4. [13] for every point $x \in H$, there exists a unique nearest point in C denoted by P_C x such that

$\|x - P_C x\| \le \|x - y\|, \quad \forall y \in C,$ (4)

Where P_C is called the metric projection of *H* onto C.

Remark1. It is well known that P_C is non-expansive mapping and satisfies

$\langle x - y, P_C x - P_C y\rangle \ge \|P_C x - P_C y\|^2, \quad \forall x, y \in H.$ (5)

Moreover, P_C x is characterized by the fact P_C x ∈ C and

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$\langle x - P_C x, y - P_C x \rangle \leq 0,$ (6)

And

$\|x-y\|^2 \geq \|x - P_C x\|^2 + \|y - P_C x\|^2, \ \ \forall x \in H, \ y \in C.$ (7)

Definition5. A multi-valued mapping $M: H \rightarrow 2H$ is called monotone if for

All $x, y \in H,\ u \in Mx$ and $v \in My$ My such that $\langle x - y, u - v \rangle \geq 0.$

Definition6. A multi-valued monotone mapping $M : H \rightarrow 2^H$ is maximal if the Graph(M), the graph of M , is not properly contained in the graph of any other monotone mapping It is known that a multi-valued monotone mapping M is maximal if and only if for $(x, u) \in H \times H,\ \langle x - y, u - v \rangle \geq 0,$ for every $(y, v) \in$ Graph(M) implies that $u \in Mx.$

Definition 7. *Let M:* $H \rightarrow 2^H$ be a multi-valued maximal monotone mapping then, the resolvent mapping $J_\lambda^M : H \rightarrow H$ associated with *M*, is defined by λ

$$J_\lambda^M(x) := (I + \lambda M)^{-1}(x), \ \ \forall x \in H,$$

For some λ > 0, where I stands identity operator on H.

Remark 2. we note that for all $\lambda > 0$, the re-solvent operator J_λ^M is single-valued, non-expansive and firmly non-expansive.

Lemma2*(Demiclosedness Principle) Assume that T is non-expansive self-mapping of a nonempty, closed and convex subset C of a Hilbert space H. If T has a fixed point, then I − T is demiclosed, i.e., whenever* $\{x_n\}$ *is a sequence in C converging weakly to some* $x \in C$ *and the sequence* $\{(I - T)x_n\}$ *converges strongly to some y, it follows that* $(I - T)x = y$*. Here I is the identity mapping on H.*

Definition8. A mapping $T: C \rightarrow C$ is said to be k-strict pseudo contractive, if there exists a constant $0 \leq k < 1$ such that

$$\|Tx - Ty\|^2 \leq \|x - y\|^2 + k\|(I - T)x - (I - T)y\|^2, \ \forall x, y \in C.$$

Lemma2. Let T: C → C be a k-strictly pseudo contractive mapping. Let γ and δ be two positive real numbers. Assume that $(\gamma + \delta)k \leq \gamma$. Then

$$\|\gamma(x - y) + \delta(Tx - Ty)\| \leq (\gamma + \delta)\|x - y\|. \qquad (8)$$

Lemma3. Let $T: C \to C$ be a k-strict pseudo contractive mapping the

(1) T satisfies the Lipschitz condition

$$\|Tx - Ty\| \leq \frac{1+k}{1-k}\|x - y\|, \quad \forall x, y \in C; \qquad (9)$$

(2)The mapping I − T is Semi closed at 0;

(3) The set Fix (T) of T is closed and convex so that the projection $P_{FIX(T)}$ is well defined.

Definition9. A mapping $T: H \to H$ is said to be averaged if and only if it can be written as the average of the identity mapping and a non-expansive mapping, i.e.,

$$T = (1 - \alpha)I + \alpha S$$

where $\alpha \in (0, 1)$ and S : H → H is non-expansive and I is the identity operator on H We note that the firmly non-expansive mappings (in particular, projections on nonempty, shut and convex subsets and resolvent operators of maximal monotone operators) are averaged. Obviously, averaged mapping is a non-expansive mapping the accompanying are some key properties of averaged mappings.

Proposition 1 *(i) If* $T = (1 - \alpha)S + \alpha V$*, where* $S : H \to H$ is averaged,

$V: H \to H$ is non-expansive and $\alpha \in (0, 1)$, then T is averaged;

(i) The composite of finitely many averaged mappings is averaged;

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(ii) If the mappings $\{T_i\}_{i=1}^N$ are averaged and have a nonempty common fixed point, then $\bigcap_{i=1}^{N} \mathrm{Fix}(T_i) = \mathrm{Fix}(T_1T_2...T_N);$

(iii) If T is τ -ism, then for $\gamma > 0, \gamma T$ is $\frac{\tau}{\gamma}$-ism;

(iv) T is averaged if and only if, its complement $I - T$ is τ -ism for some $\tau > \frac{1}{2}$.

Definition10. A family $S: = \{T(s): 0 \leq s < \infty\}$ of mappings from C into itself is called non-expansive semi-group on C if it satisfies the following conditions:

(i) $T(0)x = x$ *for all* $x \in C$;

(ii) $T(s+t) = T(s)T(t)$ *for all* $s, t \geq 0$;

(iii) $\|T(s)x - T(s)y\| \leq \|x - y\|$ *for all* $x, y \in C$ *and* $s \geq 0$;

(iv) for all $x \in C$, $s \mapsto T(s)x$ *is continuous.*

The set of all the common fixed points of a family S is denoted by Fix(S), i.e.,

$$\mathrm{Fix}(S) := \{x \in C : T(s)x = x, 0 \leq s < \infty\} = \bigcap_{0 \leq s < \infty} \mathrm{Fix}(T(s)), \tag{10}$$

Where Fix (*T (s))* is the set of fixed points of *T (s).* It is well known that *Fix(S)* is closed and convex.

Lemma4. *Let C be a nonempty, bounded, closed and convex subset of a Hilbert space H and let S:* = $\{T(s) : 0 \leq s < \infty\}$ *be a non-expansive semi-group on C. Then for t > 0 and for every 0 ≤ h < ∞,*

$$\lim_{t \to \infty} \sup_{x \in C} \left\| \frac{1}{t} \int_0^t T(s)x ds - T(h)\left(\frac{1}{t}\int_0^t T(s)x ds\right) \right\| = 0.$$

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Definitio11. An operator *B: H → H* is said to be strongly positive bounded linear operator, if there exists a constant $\bar{\gamma} > 0$ such that

$$\langle Bx, x\rangle \geq \bar{\gamma}\|x\|^2, \qquad \forall x \in H.$$

Lemma5. Assume that B is a strongly positive self-adjoint bounded linear operator on a Hilbert space H with constant $\bar{\gamma} > 0$ and $0 < \rho \leq \|B\|^{-1}$. $Then\ \|I - \rho B\| \leq$

Lemma6. *Let C* be a nonempty, closed and convex subset of a real Hilbert space *H, let f : H → H* be an α-contraction mapping and let B be a strongly positive self-adjoint bounded linear operator with constant $\bar{\gamma}$. Then for every $0 < \gamma < \frac{\bar{\gamma}}{\alpha}$, $(B - \gamma f)$ is strongly monotone with constant $(\bar{\gamma} - \gamma\alpha)$, *i.e.,*

$$\langle x - y, (B - \gamma f)x - (B - \gamma f)y\rangle \geq (\bar{\gamma} - \gamma\alpha)\|x - y\|^2.$$

$$\langle x - y, (B - \gamma f)x - (B - \gamma f)y\rangle \geq (\bar{\gamma} - \gamma\alpha)\|x - y\|^2.$$

Definition12. Let C be a nonempty subset of a Hilbert space H and let $\{x_n\}$ be a sequence in H. Then $\{x_n\}$ is Fejer monotone with respect to C if

$$\|x_{n+1} - x\| \leq \|x_n - x\|, \quad \forall x \in C.$$

Variational inequalities, equilibrium problems and iterative methods

In this segment, we give brief survey of a few classes of variational inequalities and equilibrium problems. Further, we give brief survey of some iterative methods for tackling settled point problems, variational inequalities and equilibrium problems.

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1 Variational inequalities

Let a $a(\cdot,\cdot): H \times H \to \mathbb{R}$ be a bilinear form.

Problem1. For given $f \in H^*$, *find* $x \in C$such that

$$a(x, y-x) \geq \langle f, y-x \rangle, \quad \forall y \in C. \qquad (1)$$

The inequality (1) is termed as variational termed which characterizes the classical Signorine issue of elasto-statistics, that is, the analysis of a straight flexible body in contact with an inflexible grinding less foundation. This issue was investigated and examined by Lions and Stampacchia by utilizing the projection technique.

In the event that the bi direct frame is technique, at that point by Riesz-Fr'echet hypothesis, we have

$$a(x, y) = \langle A(x), y \rangle, \ \forall x, y \in H,$$

Where $A: H \to H^*$is a continuous linear operator? Then Problem 1 is equivalent to the following problem:

Problm1. Find $x \in C$ such that

$$\langle A(x), y-x \rangle \geq \langle f, y-x \rangle, \ \forall y \in C. \qquad (2)$$

If $f \equiv 0 \in H^*$,Then (2) reduces to the following classical variational inequality problem introduced by Hartmann and Stampacchia.

Problem3. Find $x \in C$ such that

$$\langle A(x), y-x \rangle \geq 0, \ \forall y \in C. \qquad (3)$$

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The solution set of variational inequality issue VIP (3) is indicated by Sol (VIP (3)). In the variational inequality formulation, the underlying raised set C does not rely on the solution. In numerous vital applications, the arched set C likewise depends implicitly on the solution. For this situation, VIP (3) is known as semi variational inequality which emerges in the investigation of impulse control hypothesis and choice science, see for instance. Semi variational inequality was introduced and examined by Bensoussan, Goursat and Lions.To is more precise, given a multi-esteemed mapping $C : x \to C(x)$, which associates a nonempty, shut and arched subset $C(x)$ of H for every x ∈ H, a typical semi variational inequality issue is:

Problem4. Find $x \in C(x)$ such that

$$a(x, y - x) \geq \langle f, y - x \rangle, \quad \forall y \in C(x). \tag{4}$$

In many important applications, see for example Baiocchi and Capelo, Bensoussan and Lions and Mosco, the underlying set $C(x)$ is of the following form:

$$C(x) = K + m(x),$$

where m : $H \to H$ is a nonlinear mapping and K is a nonempty, shut and curved subset in H. Note that if the m is a zero mapping, at that point Problem 4 is same as Problem 1.

Further, since the general issue of equilibrium of flexible bodies in contact with unbending bodies on which frictional powers are created is a standout amongst the most difficult problems in strong mechanics. Duvaut and Lions investigated the accompanying variational inequality problem with friction:

Problem5. For $f \in H^*$, *find* $x \in C$ such that

$$a(x, y - x) + \psi(y) - \psi(x) \geq \langle f, y - x \rangle, \ \forall y \in C, \tag{5}$$

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Where $\psi : H \to \mathbb{R} \cup \{+\infty\}$ is a legitimate, convex and bring down semi continuous functional. Issue characterizes the classical Signorine issue with frictional power. The existence of its solution has been demonstrated by Glowinski et al and Neˇcas et al. The entire investigation of boundary esteem issue emerging in the detailing of Signorine issue with rubbing is a fascinating issue both in mechanics and numerical perspectives. A generalization of the Problem 5 is the accompanying:

Problem6. Given f ∈ H*, find x ∈ C such that

$$a(x, y - x) + \phi(x, y) - \phi(x, x) \geq \langle f, y - x \rangle, \quad \forall y \in C, \qquad (6)$$

Where $\phi : H \times H \to \mathbb{R}$ is an appropriate nonlinear frame? This kind of issues has been examined in Duvaut and Lions Kikuchi and Oden the Problem 6 characterizes the liquid move through porous media and Signorini issues with non-nearby grindings. For physical and mathematical plan of the inequality (6), see for instance Oden and Pires for related work; see likewise Baiocchi and Capelo and Crank

From that point forward various generalizations of the previously mentioned variational disparities have been presented and considered by various creators. Some of them are given beneath:

In 1975, Noor stretched out the Problem 2 to contemplate a class of somewhat nonlinear elliptic boundary esteem issues having constraints. Given nonlinear operators T, A: $H \to H^*$, Noor considered the accompanying issue:

Problem7. Find $x \in K$ such that

$$\langle T(x), y - x \rangle \geq \langle A(x), y - x \rangle, \quad \forall y \in K. \qquad (7)$$

Then inequality (7) is known as mildly nonlinear variational inequality.

Problem8. Find $x \in C$ such that

$$\langle T(x), y - x \rangle + \psi(y) - \psi(x) \geq \langle A(x), y - x \rangle, \quad \forall y \in C. \qquad (8)$$

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Problem8 has been studied by Siddiqi et al.in the setting of Banach space.

Problem9. Find $x \in C$ such that

$$\langle T(x), y - x\rangle + \phi(x, y) - \phi(x, x) \geq 0, \quad \forall y \in C. \tag{9}$$

Problem 9 has been studied.

CONCLUSION

An essential generalization of variational disparity is an arrangement of variational imbalances (to put it plainly, SVIP). In 1971, the arrangement of variational imbalances emerging in film issue. The Nash harmony issue for differentiable capacities can be planned as a variational disparity issue over product of sets .Various issues emerging in operation research, economics, diversion theory, mathematical physics and different ranges can likewise be consistently displayed as a variational imbalance issue over product of sets. The first variational disparity issue defined on the product of sets into an arrangement of variational imbalances, which is anything but difficult to settle, to build up some arrangement strategies for variational disparity issue over product of sets. Later it was discovered that these two issues are proportionate. From that point forward various researchers considered the presence and iterative approximations of arrangements of different systems of abstract variational disparities.A broader outcome than that in was set up by Brezis, Nirenberg and Stampacchia.Equilibrium issue (to put it plainly, EP) and concentrated its reality hypothesis. The set of solutions of EP (3) is meant by Sol (EP (3)). From that point forward equilibrium issues have been expanded and summed up in a few headings utilizing novel and creative systems both for their own particular purpose and for applications. It is realized that the equilibrium issue has an extraordinary effect and influence in the development of a few themes of science and engineering. It worked out that the theories of many surely understood issues could be tted into the theory of equilibrium issues.

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