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AN EXAMINATION OF SOME GEOMETRICAL AND ANALYTICAL STARLIKENESS OF SECTIONS OF UNIVARIATED FUNCTION CLASSES

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ABSTRACT

Complex analysis has a number of subfields, one of which is called Geometric Function Theory. This subfield works with and researches the geometric features of analytic functions. To put it another way, geometric function theory is a branch of mathematics that is distinguished by an unusual marriage between geometry and analysis. In this article, we'll look at analytical functions that are defined on the complex plane C's open unit disc D=z:|z|1. With the normalization f(0) = 0 = f0(0) 1, let LU stand for the family of all locally univalent map-pings fanalytic in D. S stands for the subfamily of univalent mappings. Some significant and well-known standard subclasses of S are the class of convex, starlike of order 1/2, and close-to-convex mappings, designated by C, S, S(1/2), and K, respectively. However, it is not possible to analyze the class of these functions as a whole for a specific category of issues. Within the scope of this paper, we will investigate the issues that arise with several new kinds of functions that are connected to univalent functions. In this paper we discuss about some classes of analytic and univalent functions and Geometrical and Analytical Phenomena Related to Univalent Function Classes

KEYWORDS: *Univalent, Function, Analytical, Geometrical, etc.*

1. INTRODUCTION

When the Bieberbach hypothesis regarding the size of the moduli of the Taylor coefficients of these functions was made in 1916, studies on analytical univalent functions became the focus of extensive research. The size of these functions' moduli was the subject of this speculation. The endeavour to discover a solution to the hypothesis led to the development of a number of spectacularly original mathematical techniques that later had a great impact. These techniques include, for instance, Lowner's parametric representation approach, the area method, Grunsky inequalities, and methods of variations. Despite de Branges's definitive debunking of the concept in the year 1985, complex function theory has remained a very active and important area of research.

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The theory of multivalent and univalent functions has long been a well-established field of study. The name "Geometric Function Theory" refers to the intriguing way that the theory demonstrates how geometry and analysis interact with one another. This theory has drawn a lot of mathematicians and other scholars over the past few decades since there are so many distinct avenues that study may go in. An examination of a section of the geometric function theory is the focus of this paper. The geometrical and analytical characteristics of particular classes that are related to univalent functions are the focus of this area of the theory.

A function f is said to be univalent in a domain D if and only if it accepts no more than one value in D and is analytic in D with the exception of no more than one simple pole. In other words, we had to have

$$F(z_1) \neq f(z_2)$$

$$\text{if} \quad z_1, z_2 \in D \quad \text{and} \quad z_1 \neq z_2$$

As a result, f is univalent in D if it maps D onto a schlicht (a word from German) or simple domain, that is, a domain without branch points and without self-overlapping. Due of this, the univalent functions are also referred to as schlicht or simple functions.

To be termed simple, the theory of univalent functions is considerably too extensive and complex. As a result, in order to make the study task more doable, it is essential important to settle on a few simplifying hypotheses. The most straightforward method is to replace a given domainD with the unit discU=z: l. This strategy is the simplest. One advantage it provides is the capability to represent functions as power series, which facilitates calculation and results in formulas that are clear and appealing. In reality, any simply linked domain with a boundary made up of more than one point can be mapped onto the unit disc in accordance with the Riemann Mapping Theorem. This has been proven to be a fact. The characteristics of the original function can easily be translated into the characteristics of any univalent function in Dis that is connected to a univalent function in U if the function that maps D onto U is known. This only functions, though, if the function that maps D onto U is known.

Additionally, a function f that is analytic and univalent in the unit disc U is normalised by the circumstances by which it is determined

$$f(O) = 0 \text{ and } f'(O) = 1$$

Indeed, if f is univalent, then the function is also univalent

$$g(z) = \frac{f(z) - f(0)}{f'(0)}$$

and it is possible to directly translate any property of the function g into a comparable property of the function f. Because the derivative of an analytic univalent function does not disappear, the division by $f'(O)$ is allowed to take place, as it may be shown out here. A normalized function, denoted by$f$, will therefore have the following power series expansion:

$$f(z) = z + a_2 z^2 + \cdots . a_n z^n + \cdots,$$

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i.e, $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$

Let us designate by the letter A the class of functions f that have theequ. form, and which are able to be shown to be analytic in the unit disc $U = \{z: |z| < l\}$. Denote by the letter S the class consisting of all the functions in A that are equivalent to one another in the variable$U$.

The class of univalent functions with one simple pole in U is normalised by demanding that the pole be located at the origin and that the residue of the pole have the value 1. In point of fact, if the pole of function f is located at the point z_0 that is part of the unit U and the residue of the pole has the value a, then we consider the function g provided by instead of function$f$

$$g(z) = \frac{1 - |z_0|^2}{a} f\left(\frac{z + z_0}{1 + \overline{z_0 z}}\right)$$

It is obvious that g is univalent in U because its pole is at the origin and its pole residue has the value 1. The letter Σ stands for the class of functions that can be normalized in this way. Therefore, the functions in Σ have the Laurent series expansion of the form:

$$f(z) = \frac{1}{z} + a_0 + a_1 z + a_n z^n + \ldots + a_n z^n + \cdots$$

$$f(z) = \frac{1}{z} + \sum_{n=2}^{\infty} a_n z^n$$

In 1907, Koebe published a paper titled "The Uniformization of Algebraic Curves," which is considered to be the foundational work for the study of univalent functions. A large number of modern mathematicians became interested in Koebe's discoveries and the challenges he posed. In the year 1916, it was Bieberbach who determined that a particular constant known as Koebe's constant had the exact value of 1/4 as its value. Only the existence of this constant was able to be demonstrated by Koebe. In addition, Bieberbach demonstrated that if a function

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

if this is the case and U is analytic and univalent, then

$$|a_2| \leq 2,$$

and the equality only takes place for the functions.

$$f(z) = \frac{z}{(1 - \eta z)^{2'}} |\eta| = 1$$

This encouragedBieberbach to conjecture

$$|a_n| \leq n, \qquad \forall n = 2,3,4, \ldots$$

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The failure to settle the Bieberbach conjecture in its general form resulted in the development of a number of significant and powerful approaches, such as the variational techniques, the inequality techniques of Goluzin and Grunsky, and the differential equation techniques developed by Lowner. Numerous scholars have looked into and looked into various subclasses of the classes of univalent and multivalent functions. These researchers have looked into and looked into these subclasses. More than 8,000 research articles have been published so far in this topic, and they provide a clear indication of both the purpose and breadth of research in the theory of univalent and multivalent functions.

2. THEORY OF UNIVALENT FUNCTION

There are numerous crucial branches in complex analysis. The Univalent Function Theory is one of the main branches, which examines one-to-one analytic functions in the unit disc $D = \{z: |z| < 1\}$normalized to have Taylor series$f(z) = z + a_2z^2 + a_3z^3 + ..$

A function is said to be normalized if $f(0) = 0$and$f'(0) = 1$. S is the name given to the category of functions that satisfy the requirements of being analytic, univalent, and normalised. In common parlance, the analytic and univalent function is most commonly referred to as conformal mapping. The many different analytical and geometrical features of the functions that fall under the class S have been the subject of a substantial amount of academic inquiry. A well-known conclusion is known as Bieberbach's Conjecture, which was formulated in 1916 and states that the Taylor coefficients must meet the condition $|a_k| \leq k$ for every given value of f in S, where k might take the values 2, 3, 4, etc.... A large number of researchers have made efforts to find an answer to the conjecture. Only a few numbers of people were able to produce a partial proof for specific subclasses of S or gain the proof for specific values of k. The theory of univalent functions has been enriched in numerous different areas as a result of an attempt to address the conjecture. It wasn't until 1985 that de Branges presented a convincing evidence of his theory.**[A. W. Goodman, 1983]**

The Omitted Area Problem is another well-known problem in the field of univalent functions. It was first posed in 1949 by A.W.Goodman and asks the following question: "what is the maximum area $A*$ of the unit disc D that can be omitted by the image of the unit disc under a univalent normalised function?" (What is the largest area that can be omitted by the image of the unit disc?) A significant number of researchers have focused their attention on finding a solution to this issue and establishing limitations for$A*$.

The fact that it is so difficult to manipulate tangible instances constitutes the primary roadblock on the path to a successful resolution of the problem. In the year 1907, Koebe laid the groundwork for what would become known as the theory of univalent function. There are several different existence theorems for canonical conformal mappings, and they all start with the traditional Riemann Mapping Theorem. On the other hand, there is a comprehensive theory of the qualitative features of conformal mappings, which focuses mostly on prior estimates and is referred to as the distortion theorems (including the Bieberbach conjecture with the proof of the De-Branges). Throughout the course of our research, we look into both univalent and multivalent functions in great depth. The subclasses of these functions, such as starlike, convex, close to convex, spiral like, pre-starlike, typical real functions, and Bazilevicfunctions, are defined on the unit disc. $U = \{z: |z| < 1\}$ and $U* = \{z : 0 < |z| <$

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1}bounds, the radius of starlikeness, convexity and being close to being convex, extreme points, the zone of univalency, convex linear combination, and other similar concepts.

The primary importance is in the promotion of interdisciplinary work among pure mathematicians and the development of novel linkages between analytical analysis, applied mathematics, and geometry. We have placed a significant amount of emphasis on the linkages between the omitted area problem and conformal mappings in the unit disc. Second, multidisciplinary knowledge that has led to a better understanding of the phenomenon has been armed with materials and methods that can handle any assignment in a manner that is more scientific. It has come to our attention that the univalent function theory is relevant to a large number of subfields within mathematics and has various applications in the field of engineering. Numerous mathematicians are employing the theory in order to find answers to issues that have arisen in the fields of engineering and technology. The comprehension of the findings of this study is very interesting, both from a theoretical and an applied point of view.

The "Bieberbach Conjecture" is just one example of the numerous mathematical conjectures that have been resolved thanks to the use of the theory of geometric functions. This hypothesis has been resolved for some values of n and for all values of n for certain subclasses of univalent functions; nonetheless, the conjecture in its whole has not yet been resolved and is still up for debate.

- **Open conjectures:**

1. If f and g are in S (normalised univalent function in E), then f.g must be in S as well. If this conjecture were correct, it would be simple to demonstrate (in numerous ways) that $|a_n|\leq n$, for every function in S. Unfortunately, hypothesis 2 is untrue and has been disproven on multiple times.

2. Goodman proposed that if f and g are in CV (Let CV be the set of all normalised univalent functions $f(z)$ for which $f(E)$ is a convex region, and when f(z) is univalent in E, we say that the domain $D = f(E)$ is a simple domain), then $(f + g)/2$ is at most 2-valent. Styer and Wright created a pair of functions in CV for which $(f + g)/2$ is 3-valent and they venture the judgement that this sum "may very well be infinite-valent for some f and g in CV".

- The following Ruscheweyh and Salagean operators are provided:

1. The differential operator Ruscheweyh of order n is

$$D^n f(z) = \frac{z\left(z^{n-1} f(z)\right)^{(n)}}{n!}, n\epsilon N_0 = NU\{0\}$$

Consider that $D^0 f(z) = f(z) \; and \; D' f(z) = zf$

2. Salagean has introduced the operator known as the Salagean operator.

$$D^0 f(z) = f(z), D^1 f(z) = Df(z) = zf'(z);$$

$$D^n f(z) = D\left(D^{n-1} f(z)\right); n\epsilon N = \{1,2,3..\}$$

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Consider that $D^n f(z) = z + \sum_{k=2}^{\infty} {}_{k^n} a_k z^k$, $n \in N_0 = NU\{0\}$

3. BASIC SUBCLASSES OF UNIVALENT FUNCTIONS

In this part, the definitions of several fundamental subclasses of these analytic univalent functions are given in terms of simple geometric features. Because of their strong relationship with functions that have a positive real part and with subordination, the classes in question can be completely described by using simple inequality.

It is said that a set D in the plane is starlike with respect to w_0, an interior point of D, if the type of thing that each ray that begins at w0 and intersects with D's interior in a set is either a line segment or a ray. We say that a function f is starlike with regard to w_0 if it transfers the domain $\mathbb{U}$ onto a starlike domain. When the variable w_0=0, we refer to the function f as a starlike function. We will now present an analytical characterization for such functions. A function f belonging to the class $\mathcal{A}$ is said to have starlike behaviour in $\mathbb{U}$ if and only if

$$\mathcal{R}(\frac{zf\prime(\mathcal{Z})}{f(\mathcal{Z})})>0, \mathcal{Z} \in \mathbb{U}$$

The collection of all in $\mathbb{U}$ starlike functions is denoted by S*. Alexander (1915) started out looking into this class.A set D in the plane is referred to as convex if the line segment connecting any two locations w_1 and w_2 inside of D is also inside$D$. A function is said to be convex in $\mathbb{U}$ if it maps another function, $\mathbb{U}$, onto a convex domain. In other words, a convex domain is one that, with regard to each of its points, has the shape of a star. The analytical description of the convex function is provided by

$$f \in \mathcal{K} \Leftrightarrow \mathcal{R}\,(\frac{zf''(\mathcal{Z})}{f'(\mathcal{Z})}>0, z \in \mathbb{U}$$

where K is the collection of all logical operations that are convex in $\mathbb{U}$. For example, the functions log $[\frac{1+z}{1+z}]$ and $\frac{z}{1-z}$are convex in U. From the arguments above, it is clear that

$$K \subset S * \subset S$$

Although not convex, the Koebe function resembles a star. Alexander (1915) was the first to discover that convex and starlike functions have a strong analytic link; this finding is known as Alexander's Theorem. If f is an analytical function in $\mathbb{U}$ with $f(0) = 0$ and $'(0) = 1$, then, according to this,

$$f \in \mathcal{K} \Leftrightarrow zf' \in s *, z \in \mathbb{U}$$

The classes $S*(a)$ and $K(a)$ of starlike and convex functions of order$a, 0 \leq a \leq 1$ were introduced by Robertson by means of an order language. These classes are defined by

$$S*(a) = \{f \in A: \mathcal{R}\,(\frac{zf'(\mathcal{Z})}{f(\mathcal{Z})} > (a), z \in \mathbb{U}\}$$

$$\mathcal{K}(a) = \{f \in A: zf' \in s*(a), z \in \mathbb{U}\}$$

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We obtain the well-known classes of starlike and convex univalent functions for $a = 0$. F.

4. GEOMETRIC FUNCTION THEORY

The area of complex analysis known as "Geometric Function Theory" deals with and investigates the geometric characteristics of analytical functions. In other words, the field of mathematics known as Geometric Function Theory is distinguished by an unusual union of geometry and analysis. Despite having nineteenth-century roots, it continues to find novel applications today. The study of univalent function qualities is a major focus of geometric function theory of a single-valued complex variable. If the image domain of the open unit disc $U = z\,C: |z| < 1$ under a univalent function has some appealing geometric characteristics, it may be of interest. A domain with intriguing features is best shown by a convex domain. One that is star-shaped in relation to a point is another illustration of such a domain.

It is a traditional subject to study geometric function theory. However, it continues to find new uses in a wide range of domains, including non-linear integrable systems theory of partial differential equations, current mathematical physics, and more conventional branches of physics like fluid dynamics. The history of geometric function theory is not as long as those of other areas of mathematics. Function theory began to take off in the 18th century with the work of Euler. The nineteenth century saw the development of modern function theory. Function theory saw quite significant victories in a relatively short period of time throughout the previous century.

The methods of algebraic geometry and function theory on compact Riemann surfaces have found relevance in constructing 'finite-gap' solutions to non-linear integrable system, which is a growing, specialised area of mathematics that has many connections to mathematical physics. As a result, there has been a rebirth in interest in geometric function theory over the past few decades. For the computation of so-called Veneziano amplitudes, early string theory models rely on aspects of geometric function theory. Even recent advancements in the constructive approach to linear and non-linear boundary value and initial value issues using spectral analysis are likely to lead to a role for geometric function theory in the solution of a wide variety of partial differential equations. Classical work has been done in the field of geometric function theory. However, it continues to find new applications in an ever-growing array of topics, such as current mathematical physics, more traditional branches of physics such as fluid dynamics, and the theory of non-linear integrable systems and partial differential equations.

In comparison to other subfields of mathematics, the geometric function theory is a more recent development. Euler is credited for initiating the initial stirrings of function theory in the 18th century. The nineteenth century was a pivotal time for the development of modern function theory. The field of function theory achieved a considerable deal of success within a relatively brief period of time over the previous century. In the space of just a few decades, a building of academic significance was constructed, and it almost instantly received the greatest appreciation from the mathematical community.

The theory of univalent functions is considered to be one of the most aesthetically pleasing aspects of geometric function theory. Aside from the Riemann mapping Theorem, its

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beginnings can be traced back to Gronwall's proof of the area theorem in 1914 and to Bieberbach's estimate for the second coefficient of normalised univalent functions in 1916 and its consequences. Both of these can be considered Gronwall's contributions to the field. By that point, univalent function theory had become a distinct field of study in its own right.

5. ANALYTIC FUNCTIONS OF NORMALIZED CLASS $\mathcal{A}$

At the point z_0 in $\mathbb{C}$, a complex-valued function f of the complex variable z is said to be differentiable, if $\lim_{z\to z_0}\frac{f(z)-f(z_0)}{z-z_0}$ exists. An analytic function f at z0 is differentiable at z0 and at every point in some neighbourhood of z_0. If a complex-valued function f of a complex variable z is analytic at every point in that domain D, it is said to be analytic in that domain D.

One of the wonders of complex analysis is that an analytic function f has derivatives of all orders and has a Taylor series representation of the form.

$$f(z)=\sum_{n=0}^{\infty} a_n (z-z_0)^n, a_n=\frac{f^{(n)}(z_0)}{n!}$$

convergent in some open disk at z_0. We will be primarily concerned with the class $\mathcal{A}$ of analytic functions in the open unit disc $\mathbb{U}$, normalised by the requirements $f(0) = 0$ and $f'(0) = 1$. The generality is unaffected by these normalising criteria. Specifically, $\mathcal{A}$ denotes the class of functions of the form:

$$f(z)= z+\sum_{n=2}^{\infty} a_n z^n \; z\in\mathbb{U}$$

Assume that S is a subclass of $\mathcal{A}$, the class of all analytic, univalent functions f in the open unit disc $\mathbb{U}$. Due to the Riemann Mapping Theorem, which states that any simply connected domain D $\subset\mathbb{C}$ with at least two border points may be translated conformally onto the open unit disc $\mathbb{U}$, open unit disc $\mathbb{U}$ was used above in place of an arbitrary domain D.

If $w = f(z)$ assumes different values for w, then the function f analytical in $\mathbb{U}$ is said to be univalent in $\mathbb{U}$ for distinct z in $\mathbb{U}$. In this instance, $w = f(z)$ has just one root in $\mathbb{U}$ at most. Univalent is sometimes referred to by other terms like simple or Schlicht (the German word for simple). A univalent function, or one that never accepts the same value twice, is one that has the formula $f(z_1) \neq f(z_2)$ for all points z_1 and z_2 in $\mathbb{U}$ with $z_1 \neq z_2$. $\mathcal{A}$ univalent function, or f, is a one-to-one (injective) mapping of the domain $\mathbb{U}$ onto another domain.

In the study of the class S, the univalent Koebe function $k(z)=\frac{1}{(1-z)^2}$ is crucial. The only extremal functions for different problems in the class S are the Koebe function and its rotations $e^{-ia}k(e^{-ia}z)$ and a$\in\mathbb{R}$. Bieberbach is responsible for the two main outcomes (1916) indicating that when $f \in S$, then $|a_2| \leq 2$ and $f(z)$ assumes every value of w such that |w|<1/4. These two outcomes are both precise. The Koebe function k maps the disc $\mathbb{U}$, one-one and conformally onto the w-plane cutting from -1/4 to -∞ along the negative real axis. DeBranges (1985) proved the Bieberbachconjecture for the coefficient estimates of the class S that $|a_n| < n$ holds for n ≥ 2 and so solved the famous problem in the univalent function theory.

6. ANALYTIC AND UNIVALENT FUNCTIONS

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Let $\mathcal{H}$ us refer to the class of analytic functions on the unit disc as H, where D stands for the equation D: = {z ∈ℂ: |z| < 1}. In this context, "$\mathcal{H}$" refers to a topological vector space that is locally convex and is endowed with the topology of uniform convergence across compact subsets of "𝔻." Let us designate by the letter $\mathcal{A}$ the class of functions f ∈$\mathcal{H}$ in which f (0) = 0 and f' (0) = 1, and let us designate by the letter $\mathscr{S}$ the subclass of functions f ∈$\mathcal{A}$ that are univalent (that is, one-to-one) in 𝔻. The following is the representation that applies to each function f ∈$\mathscr{S}$:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

It is stated that a set D ⊂ℂ has a starlike relationship with regard to a point z_0∈ D if the line segment that connects z ∈ D to every other point z ∈ D is wholly inside D. Convexity may be defined as the property of a set D being such that every line segment linking any two points of D falls wholly inside D. If the function f (D) is starlike with regard to the origin, then the function f∈$\mathcal{A}$is said to be starlike (or convex, respectively) (convex respectively). In the set S, the classes of starlike functions are denoted by the letter $\mathscr{S}$, while convex functions are denoted by the letter$\mathscr{S}$ * and ℂ. It is well knowledge that a function f∈$\mathcal{A}$is considered to be a member of $\mathscr{S}$ if, and only if, the value of Re (zf' (z)/f (z)) > 0 for z ∈𝔻. In the same vein, a function f∈$\mathcal{A}$is considered to be part of category $\mathcal{C}$ if and only if the value of the ratio Re (1+z f''(z)/f' (z)) > 0 for z ∈𝔻. Alexander demonstrated that f ∈ $\mathcal{C}$if, and only if, zf'∈$\mathscr{S}$ *. Given α ∈ (−π/2, π/2) and g ∈$\mathscr{S}$ *, a function f∈$\mathcal{A}$is said to be close-to-convex with argument α with respect to g if:

$$Re\left(e^{ia}\frac{zf'(z)}{g(z)}\right) > 0\ for\ z \in \mathbb{D}$$

Let the class of all functions of this kind be denoted by $\mathcal{K}_a$(g), and:

$$\mathcal{K}(g) := \bigcup_{a\epsilon(-\pi/2,\pi/2)} \mathcal{K}_a(g) and\ \mathcal{K}_a := \bigcup_{g\in\mathscr{S}^*} \mathcal{K}_\alpha(g)$$

be the groups of functions that are near to convex with respect to g, and the groups of functions that are close to convex with respect to argument α. Let

$$\mathcal{K} := \bigcup_{a\epsilon(-\pi/2,\pi/2)} \mathcal{K}_a = \bigcup_{g\in\mathscr{S}^*} \mathcal{K}_\alpha(g)$$

be the class of functions that are near to being convex. It is common knowledge that each and every function that is nearly convex is univalent in D. When it comes to geometry, f ∈$\mathcal{K}$ indicates that the complement of the picture domain f (𝔻) is the union of half-lines that do not overlap each other. The appropriate inclusions provide a connection between these standard classes. $\mathscr{C} \subsetneq \mathscr{S}^* \subsetneq \mathcal{K} \subsetneq \mathscr{S}$.

Let us assume that X is a linear topological vector space and that V ⊆ X. If there is no

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representation of the form x = ty + (1 − t)z, 0 < t < 1 as a correct convex combination of two different points y, z ∈ V, then the point x falls into the category of an extreme point of the vector space V. We will refer to the collection of points that are the most extreme in V as EV. A set V is said to have a convex hull if the smallest convex set that contains V is called the convex hull of V. The V ⊆ X that include V is what is meant when we talk about the closed convex hull, which is symbolized by the symbol $\overline{co}V$. As a result, the closure of the convex hull of V is the smallest convex set that is closed and contains V. This means that the closed convex hull of V is the smallest convex set. According to the Krein–Milman theorem [9], every compact subset of a locally convex topological vector space is included in the closed convex hull of its extreme points. This assertion was made in the context of topological vector spaces. It is said that a function f is a support point of a compact subset $\mathscr{F}$, of $\mathcal{H}$ if f $\in \mathscr{F}$,and if there exists a continuous linear functional J on $\mathcal{H}$ such that Re J is not constant on $\mathscr{F}$, and if there are two further conditions.

$$Re\, J(f) = \max\{Re\, J(g): g \in \mathscr{F},$$

Supp $\mathscr{F}$, denotes the set of all support points of a compact family $\mathscr{F}$,

Studies have been done on the support points of families of convex functions that resemble stars, as well as support points of functions that are near to being convex. In this work, have shown a necessary and sufficient condition for the harmonic Bloch mapping f to be a support point of the unit ball of the normalized harmonic Bloch spaces in $\mathbb{D}$.

For 0 < λ ≤ 1, let U (λ) be the class of functions $f \in \mathcal{A}$ satisfying

$$\left| f'(z)\left(\frac{z}{f(z)}\right)^2 - 1 \right| < \lambda \; for\; z \in \mathbb{D}$$

Each function in $\mathcal{U}$ (λ) is non-vanishing in $\mathbb{D}\backslash\{0\}$ because $f'(z)(z/f(z))^2$only has finite values. Set: =$\mathcal{U}$ (1). It is also obvious that in $\mathcal{U}$ (λ), functions are locally univalent. Furthermore, functions in $\mathcal{U}$ (λ), or $\mathcal{U}$ (λ) ⊆$\mathscr{S}$ for 0 < λ ≤ 1, have been shown by. As a result, functions in $\mathcal{U}$ are univalent, however not all of them are starlike, contrary to what one would anticipate given the similarity of their analytic representations. Since the class of starlike functions is relatively broad, this makes them intriguing since in the theory of univalent functions, it matters whether a class does not totally reside within $\mathscr{S}^*$. Ali et al. have explored the properties of meromorphic functions belonging to the class $\mathcal{U}$ (λ), as well as the integral mean problem and arc length issue for functions in $\mathcal{U}$. Despite the fact that neither $\mathcal{U} \subset \mathscr{S}^*$ nor $\mathscr{S}^* \subset \mathcal{U}$, have shown that $\overline{co}\mathcal{U} = \overline{co}\mathscr{S}^*$their major finding—that $z/(1-xz)^2 \in$ S $\mathscr{S}^* \cap \mathcal{U}$ for each x such that |x| = 1. To learn more about the class $\mathcal{U}$ (λ). Let G represent the class of satisfied functions f $\in\mathcal{A}$.

$$Re\left(1 + \frac{zf''(z)}{f'(z)}\right) > -\frac{1}{2} \qquad for\; z \in \mathbb{D}$$

$\mathcal{G}$ functions are almost convex because they are convex in one direction. Additionally, if f ∈ G and is of the form, we get the coefficient inequality shown below.

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$$|a_n| \le \frac{n+1}{2} \qquad for\ all\ n \ge 2$$

with equivalence only for the rotations of the function $g_0(z)$, where:

$$g_0(z) = \frac{z - \left(\frac{x}{2}\right) z^2}{(1 - xz)^2}, \qquad |x| = 1$$

Before we begin to demonstrate our key conclusions, we will first discuss many significant lemmas. These lemmas are going to play an essential part in the demonstration of our primary results.

Lemma 1. If, and only if, there is a sequence bn of complex numbers that meet certain conditions, then J may be considered a complex-valued continuous linear functional on $\mathcal{H}$.

$$\overline{\lim_{n\to\infty}} |b_n|^{1/n} < 1$$

They are of such a kind that:

$$J(f) = \sum_{n=0}^{\infty} b_n a_n$$

where $f \epsilon \mathcal{H}$ and $f(z) = \sum_{n=0}^{\infty} a_n z^n \ for\ |z| < 1$

Lemma 2. Let $\mathscr{F}$, be a constrained subset of $\mathcal{H}$, and let J be a continuous linear function with complex values that operates on $\mathcal{H}$. Then

$$\max\{Re\ j(f): f \epsilon\ \overline{\overline{co}}\ \mathcal{F}\} = \max\{Re\ J(f): f\ \epsilon\ \mathcal{F}\} = \max\{Re\ J(f): f \epsilon \overline{Eco}\mathcal{F}\}$$

Lemma 3.$\overline{\overline{co}}\mathcal{U}$includes all functions denoted by the letter f that have a representation

$$f(z) = \int_{|x|=1} \frac{z}{(1 - xz)^2}, \qquad |x| = 1$$

Lemma 4.$\overline{\overline{co}}\mathcal{G}$f includes all of the functions and their representations.

$$f(z) = \int_{|x|=1} \frac{z - \left(\frac{x}{2}\right) z^2}{(1 - xz)^2}\, d\mu(x)$$

where μ ∈Λ, and Λsignifies the collection of probability measures that may be applied to $\partial\mathbb{D}$. In addition, $\overline{Eco}\mathcal{G}$ is made up of the functions f that make up the form.

$$f(z) = \frac{z - \left(\frac{x}{2}\right) z^2}{(1 - xz)^2}, \qquad |x| = 1$$

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In this part, we are going to describe the collection of support points that are used by the classes.

7. CONCLUSION

As a connection between geometry and analysis, the study of subclasses of analytic and univalent functions in geometric theory and applications has garnered a broad variety of attention among function theorists in recent years. It is possible for there to be as many subclasses of analytical and univalent functions as there are scholars who are interested in the field. The major importance is in the promotion of interdisciplinary work among pure mathematicians as well as the development of novel linkages between analytical analysis, applied mathematics, and geometry. This study presents the results of an investigation into univalent functions. The theory of univalent functions is a classical topic in complex analysis and is considered to be one of the most attractive subjects in geometric function theory. This problem has been studied for a very long time. It focuses on the geometric features of analytic functions that were discovered around the turn of the 20th century. On the basis of the findings made thus far, we are able to state a number of different classes of univalent functions.

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