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AN INVESTIGATION OF HYPERGEOMETRIC SERIES CONTINUATION OF TAIL FRACTIONS

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ABSTRACT

A continuous fraction is a mathematical expression formed by expressing a number as the sum of its integer component and the reciprocal of another number, then expressing this other number as the sum of its integer component and yet another reciprocal, etc. This procedure is repeated until the desired result is achieved. When an integer is substituted with another continued fraction, the iteration or recursion results in a continued fraction with a limited number of steps. Many standard functions, such as the arctangent and the logarithm, can have the tails of the Taylor series written as continuing fractions. Some of these continuous fractions have the startling side effect of dramatically accelerating the convergence of the underlying power series. This inquiry was prompted by an unexpected observation on Gregory's series. Gregory's series. Exist are functions that can be evaluated analytically and expressed in hypergeometric form. Ramanujan's discoveries on continuing fractions are clear consequences of relationships between three-term hypergeometric series. Theirq-analogues result in a number of the continuing fractions detailed in the "Lost" notebook, including the well-known one that Andrews and others have considered. We constructed continuous fraction representations for the ratio of ordinary and fundamental hypergeometric series in this study. Using this investigation, these representations were established.

KEYWORDS: *Hypergeometric, Function, continued, Fractions, etc.*

1. INTRODUCTION

Several noteworthy aspects of continued fractions are related to the Euclidean method for integer or real number computations. Using the Euclidean method, it is possible to calculate the coefficients ai of the finite continuous fraction expressions that are closely related to any rational integer p/q of the type shows (p,q). Irrational numbers are used to represent the numerical value of an infinite continued fraction, which is the limit of a series of values for finite continued fractions obtained from an endless sequence of integers. In order to construct each finite continuing fraction that is a component of the series, it is necessary to use a finite prefix of the sequence of integers that defines the infinite continued fraction. In addition, the value of every irrational integer is a unique infinite regular continuous fraction.

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Applying the non-terminating variant of the Euclidean method to the incommensurable values and 1 yields this fraction's coefficients. This type of real number representation is called the continuous fraction representation (rational and irrational).

In this study, we will present a brief introduction of the fundamental hypergeometric series and related functions that will be discussed in this dissertation. Basic hypergeometric series have gained a great deal of significance during the past few decades as a result of its applications in a range of fields, such as additive number theory, combinatorial analysis, statistical and quantum physics, vector spaces, and more. They have provided analysts with a very valuable tool that will enable them to combine and subtotal a vast number of individual results in number theory under one roof. The "Lost" Note Book of Ramanujan, which was discovered in 1976 by G. E. Andrews, sparked a renewed interest in the previously stated functions. In 1979, the American Mathematical Monthly published a beautiful piece about the "Lost" Notebook and all of its contents. The article details the notebook's discovery and its contents. The vast body of literature on basic hypergeometric series, also known as q-hypergeometric series, has reached such a level of significance and importance that the study of these series has attained an independent, respectable status rather than merely being viewed as a generalisation of the common hypergeometric series.

2. HYPERGEOMETRIC SERIES

The hyper geometric function is a specific type of function that is applicable in a variety of contexts. The hyper geometric functions serve as the foundation for the generalization of transcendental functions of higher order. These functions have applications in numerous domains, such as theoretical physics, and are also utilized in functional computer applications, such as Maple and Mathematical. In addition, a large amount of statistical and economic distribution theory research is conducted on them. In the study of fractional calculus, hyper geometric functions are conceivable. The manner in which these functions are implemented can be determined from the data (such as fractionally integrated) of time series and other economics-related fields. Given that both unemployment and price rises are decided, this relationship looks to be significant for economists. We were able to develop the accurate cumulative distribution function by employing the Bessel function, incomplete gamma, Gauss hypergeometric function, and other relevant functions. A last remark concerning hyper geometric functions. They have now attained such a level of importance in a wide variety of mathematical applications that they are incorporated in a wide variety of computer programmes, such as Maple and Mathematical, which allow symbolic manipulations. Their ability to deliver immediate solutions to problems, in addition to their generally straightforward attitude to problems, is one of their greatest strengths. It is my genuine hope that this work has successfully demonstrated their quantitative economics potential.

2.1 Hyper geometric Series

A hypergeometric series k ck is one where c0=1 and the ratio of consecutive terms is a logical function of the summation indexk, that is, one where c0=1 and the ratio of succeeding terms is a logical function of the summation indexk.

$$\frac{c_{k+1}}{c_k} = \frac{P(k)}{Q(k)},$$

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with $P(k)$ and $Q(k)$ polynomials. In this instance, is referred to as a hypergeometric word. Hypergeometric series produce functions that are known as hypergeometric functions or, more broadly, generalized hypergeometric functions. The ratio of succeeding terms can be written if the polynomials are fully factored.

$$\frac{C_{k+1}}{C_k} = \frac{P(k)}{Q(k)} = \frac{(k+a_1)(k+a_2)....(k+a_p)}{(k+b_1)(k+b_2)....(k+b_q)(k+1)},$$

where,the factor of k+1 in the denominator is kept for the purposes of historical notation, and the generalized hypergeometric function that is produced as a result is written as

$$\begin{bmatrix} a_1 a_2 \dots a_p \\ b_1 b_2 \dots b_q \end{bmatrix}; x \Big] = \sum_{k=0} C_k x^k$$

If p=2 and q=1,the function changes into a standard hypergeometric function.

Numerous sums can be expressed as generalized hypergeometric functions by examining the ratios of successive terms in the generating hypergeometric series.

2.2 Hypergeometric Functions

$F(a, b;\ c;\ x)$ is the hypergeometric function defined as

$$F(a, b;\ c;\ x) = {}_2F_1(a, b;\ c;\ x) = F(a, b;\ c;\ x)$$

$$=1 + \frac{ab}{c}x + \frac{a(a+1)b(b+1)}{c(c+1)}\frac{x^2}{2!} + \dots.$$

$$= \sum_{n-0}^{\infty} \frac{(a)n(b)n}{(c)n}\frac{x^n}{n!} |x| < 1, c \neq 0, -1, -2 \dots.$$

Where;

2: indicates the number of parameters in the numerator.

1: refers to the quantity of parameters in the denominator.

$$y_1 = F(a, b; c; x) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)}\frac{x^n}{n!}$$

is a solution to the Hypergeometric Differential Equation.

$$x(1 - x)y'' + [c - (a + b + 1)x]y' - aby = 0$$

The alternative independent option is

$$y2 = x^{1-c}F(a - c + 1, b - c + 1;\ 2 - c;\ x) \qquad c \neq -1, -2, \dots$$

Some Features of $F(a, b;\ c;\ x)$

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$$\frac{d}{dx}F(a,b;c;x) = \frac{ab}{c}F(a+1,b+1;c+1;x)$$

The generic formula based on repeated differentiation

$$\frac{d^k}{dx^k}F(a,b;c;x)=\frac{(a)_k(b)_k}{(c)_k}F(a+k,b+k;c+k;x)k = 1,2,3 \ldots.$$

3. CONTINUED FRACTION

The continuous fraction is one of the most impressive and a powerful representation of numbers and it has a long and fascinating history. Some numbers, whose decimal expansions appear utterly random, contain astonishing symmetries and patterns when unfolded into a continuous fraction. These symmetries and patterns are embedded inside the number itself.

Even though it has been around for a long time, many people are still unfamiliar with the concept of continued fractions (CF). In point of fact, continued fractions have a wide variety of applications not only in algebra but also in other areas of study including mathematics, physics, and chemistry.

The process of expressing a particular quantity in the form of a numerator and a denominator, where each denominator is made of a numerator and a denominator, and so on, is the simplest approach to create a continued fraction. In most cases, the sum of each succeeding numerator is equal to one.

Continued fractions have been around for a very long time; in fact, they have been known since since the Euclidean procedure for determining the greatest common divisor (GCD) of two integers was developed. It was close to the year 300 B.C. at the time. Research works and papers continue to be carried out, and an enormous accumulation of applications emerge as a result; this is because of the ease with which they can be dealt with, as well as the smooth way in which the calculations involved are carried out, because all that is required are the four basic mathematical operations, namely addition, subtraction, multiplication, and division. Consequently, this is due to the fact that it is simple to deal with them

The class of expressions that a generalized continuing fraction of the form is an example of is what we mean when we talk about when we use the word "continued fraction."

$$b_0 + \frac{a_1}{b_1 + \frac{a_2}{b_2 + \frac{a_3}{b_3 + \cdots}}} = b_0 + \underset{n=1}{\overset{\infty}{K}} \frac{a_n}{b_n}$$

Where the terms may be integers, reals, complexes, or functions of these) are the most generic variety.

Although some sources cite 1655 or 1656 as the year of publication, Wallis is credited with being the first person to use the word "continued fraction" in his work Arithmetical infinitorum, which was published in 1653. The term "continued fraction" has been replaced by the more modern term "anthyphairetic ratio."

3.1 Contiguous relation

It has been shown that, regardless of which is the illustrious hamster, Φ meets the requirements for the three-term contiguous relation:

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$$\frac{g(1-h)\left(1-\frac{a}{h}\right)\left(1-\frac{aq}{h}\right)\left(1-\frac{aq}{gb}\right)\left(1-\frac{aq}{gc}\right)\left(1-\frac{aq}{gd}\right)(1-\frac{aq}{ge})(1-\frac{aq}{gf})}{\left(1-\frac{hq}{g}\right)}$$

$$\times [\Phi(g-,h+)-\Phi]$$

$$\frac{h(1-g)\left(1-\frac{a}{g}\right)\left(1-\frac{aq}{g}\right)\left(1-\frac{aq}{hb}\right)\left(1-\frac{aq}{hc}\right)\left(1-\frac{aq}{hd}\right)(1-\frac{aq}{he})(1-\frac{aq}{hf})}{\left(1-\frac{gq}{h}\right)}$$

$$\times [\Phi(h-,g+)-\Phi]$$

$$-\frac{aq}{h}\left(1-\frac{h}{g}\right)\left(1-\frac{gh}{aq}\right)(1-b)(1-c)(1-d)(1-e)(1-f)\Phi=0$$

It is important to take note that we are using the notation Φ(g−, h+) to signify a "Φ" with g being replaced by g/q and h being replaced by hq.

The demonstration of this contiguous relation is based on extending the contiguous relations determined for a terminating to nonterminating complementary pairs Φ. This allows the contiguous relations to be extended to complementary pairs that do not terminate. This is made feasible by replacing the transformation with the transformation of (Gasper and Rahman 1990), which allows this to happen.

3.2 Simple Continued Fraction

Definition 1:A Simple Continued Fraction is the form of the fraction

$$a_0+\frac{1}{a_1+\frac{1}{a_2+\frac{1}{a_3+\cdots}}}$$

Where$a_i$since i > 0 and a_0 can be any number, ai are non-negative integers.**[H. Wall, 1948]**

The expression above is difficult to write and is typically written in either of these two ways:

$$a_0+\frac{1}{a_1+}\frac{1}{a_2+}\frac{1}{a_3+}$$

or by utilizing list notation

$$[a_0, a_1, a_2, a_3, \ldots]$$

to have the same meaning as the previous continued fraction.

Example 1: 43/19 = [2, 3, 1, 4]

This notation includes

$$[a_0]=\frac{a_0}{1}$$

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$$[a_0, a_1] = a_0 + \frac{1}{a_1} = \frac{a_0 a_1 + 1}{a_1}$$

$$[a_0, a_1, a_2, \ldots, a_n] = a_0 + \frac{1}{[a_1, a_2, \ldots, a_n]} = [a_0, [a_1, a_2, \ldots, a_0]]$$

Generally speaking, we have

$$[a_0, a_1, a_2, \ldots, a_n] = [a_0, a_1, \ldots a_{m-1}, [a_m, a_{m+1}, \ldots, a_n]], \text{ for } 1 \leq m \leq n$$

3.3Convergents

Definition 2: We call $[a_0, \ldots, a_m]$ (for $0 \leq m \leq n$) the m_{th} convergent to $[a_0, \ldots, a_n]$.

Example 2: In our instance, the convergents are

$$2 = \frac{2}{1}$$

$$2 + \frac{1}{3} = \frac{7}{3}$$

$$2 + \frac{1}{3 + \frac{1}{1}} = \frac{9}{4}$$

$$2 + \frac{1}{3 + \frac{1}{1 + \frac{1}{4}}} = \frac{43}{19}$$

4. HYPERGEOMETRIC SERIES AND CONTINUING FRACTIONS

Since the time of Euler and Gauss, continuing fractions have played a crucial role in Number Theory and Classical Analysis. Ramanujan made significant contributions to the theory of continuous fraction expansions, a branch of mathematics.

In Chapter 12 of his second notebook and also in his "lost" notebook, Ramanujan wrote down a number of continuing fraction identities. This particular feature of Ramanujan's work has been the subject of a number of authors' efforts to improve and expand upon it.

4.1 Definitions and Notations

We define the real or complex value of α;

$$[\alpha]_n = \alpha(\alpha + 1)(\alpha + 2)\ldots(\alpha + n + 1)\, n > 0$$

$$[\alpha]_0 = 1$$

and

$$[\alpha_1\alpha_2\alpha_3\ldots\alpha_r]_n = [\alpha_1]_n[\alpha_2]_n[\alpha_3]_n\ldots[\alpha_r]_n$$

The typical hypergeometric series is described as follows:

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$$ {}_rF_s\begin{bmatrix} a_1, a_2, \dots, a_r; z \\ b_1, b_2, \dots, b_s \end{bmatrix} = \sum_{n=0}^{\infty} \frac{[a_1, a_2, \dots, a_r]n z^n}{[b_1, b_2, \dots, b_s]n^{n!}} $$

where $r \leq s$ or $r = s + 1$ and $|z| < 1$ for the series' convergence.

As well, we specify what q-shifted factorial is.; for α and q real or complex ($|q| < 1$) we describe

$$[\alpha; q]n = (1 - \alpha)(1 - \alpha q)\dots(1 - \alpha q^{n-1}), n > 0$$

$$[a; q]_0 = 1$$

$$[a; q]\infty = \prod_{n=0}^{\infty}(1 - aq^n)$$

and $[\alpha_1, \alpha_2, \dots, \alpha_r;\ q]_n = [\alpha_1;\ q]_n, [\alpha_2;\ q]_n, \dots, [\alpha_r;\ q]_n$

As a result of the notations above, we define the generalized fundamental hypergeometric series by

$$ {}_r\Phi_s\left[\begin{matrix} a_1 & a_2 \\ b_1 & b_{2\dots}b_s \end{matrix} \dots a_r; q; z\right] = \sum_{n=0}^{\infty} \frac{[a_1, a_2 \dots a_r; q]n Z^n}{[b_1, b_2 \dots b_s; q]n} $$

where max. $(|z|, |q|) < 1$in order for the series to convergence.

4.2 Formulae involving Continued Fraction and Hypergeometric Function

$$ \frac{1}{\left[1 + \frac{z^2}{3+\frac{4z^2}{5+\frac{9z^2}{7+\frac{16z^2}{9+\dots}}}}\right]^2} = \frac{\left[z - \frac{z^3}{3+\frac{9z^2}{5+\frac{4z^2}{7+\frac{25z^2}{9+\frac{16z^2}{11+\dots}}}}}\right]^2}{z^2} = \frac{1}{\left[1 + z^2 + -\frac{2z^2}{3-\frac{2z^2}{5(1+z^2)-\frac{12z^2}{7-\frac{12z^2}{9(1+z^2)+\dots}}}}\right]^2} $$

$$ = \frac{1}{\left[1 + \frac{z^2}{1+z^2-2(-1+z^2)+\frac{9z^2}{1+z^2-4(-1+z^2)+\frac{25z^2}{1+z^2-6(-1+z^2)+\frac{49z^2}{1+\dots+z^2-8(-1+z^2)}}}}\right]^2} = $$

$$ F^{1:1;2}_{1:0;1}\left(\begin{matrix} [1{:}1,1]{:}\,[1{:}1];[\tfrac{1}{2},1],[1{:}1]; \\ [2{:}1,1]{:}\,-;[\tfrac{3}{2}{:}1] \qquad ; \end{matrix} - z^2, -z^2\right) $$

4.2 Key Results

The results that we will establish in this section are as follows:

$$ \frac{2F_1[a + i, \beta + i + 1; \gamma + i; x]}{2F_1[a + i + 1, \beta + i; \gamma + i; x]} $$

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$$= 1 - \cfrac{(\beta-\alpha)x/(\gamma+i)}{1 - \cfrac{(\gamma-\beta)(\alpha+i+1)x/(\gamma+i)(\gamma+i+1)}{1 - \cfrac{(\gamma-\alpha)(\beta+i+1)x/(\gamma+i+1)(\gamma+i+2)}{1 - \cfrac{(\gamma-\beta+1)(\alpha+i+2)x/(\gamma+i+2)(\gamma+i+3)}{1 - \cfrac{(\gamma-\alpha+1)(\beta+i+2)x/(\gamma+i+3)(\gamma+i+4)}{1 - \ddots}}}}}$$

And

$$\frac{2\Phi_1[aq^i,\beta q^{i+1};\gamma q^i;x]}{2\Phi_1[aq^{i+1},\beta q^i;\gamma q^i;x]}$$

$$= 1 - \cfrac{(\alpha-\beta)x/(1-\gamma q^i)}{1 - \cfrac{(\beta-\gamma)(1-\alpha q^{i+1})x/(1-\gamma q^i)(1-\gamma q^{i+1})}{1 - \cfrac{(\alpha-\gamma)(1-\beta q^{i+1})x/(1-\gamma q^{i+1})(1-\gamma q^{i+2})}{1 - \cfrac{(\beta-\gamma q)(1-\alpha q^{i+2})x/(1-\gamma q^{i+2})(1-\gamma q^{i+3})}{1 - \cfrac{(\alpha-\gamma q)(1-\beta q^{i+2})x/(1-\gamma q^{i+3})(1-\gamma q^{i+4})}{1 - \ddots}}}}}$$

Proof:

$${}_2F_1[\alpha + i + 1, \beta + i; \gamma + i; x] - {}_2F_1[\alpha + i, \beta + i + 1; \gamma + i; x]$$

$$= \frac{(\beta-\alpha)x}{(\gamma+i)} {}_2F_1[\alpha + i + 1, \beta + i + 1; \gamma + i + 1; x]$$

At this time, we have

$$\frac{2F_1[\alpha+i+1,\beta+i;\gamma+i;x]}{2F_1[\alpha+i,\beta+i+1;\gamma+i;x]} = 1 - \frac{(\beta-\alpha)x(\gamma+i)}{\frac{2F_1[\alpha+i,\beta+i+1;\gamma+i;x]}{2F_1[\alpha+i+1,\beta+i+1;\gamma+i+1;x]}}$$

Again,

$$2F_1[\alpha + i + 1, \beta + i+1; \gamma + i+1; x] - 2F_1[\alpha + i + 1, \beta + i; \gamma + i; x]$$

$$= \frac{(\gamma-\beta)(\alpha+i+1)x}{(\gamma+i)(\gamma+i+1)} 2F_1[\alpha + i + 2, \beta + i+1; \gamma + i+2; x]$$

that provides

$$\frac{2F_1[\alpha + i + 1, \beta + i+1; \gamma + i+1; x]}{2F_1[\alpha + i + 1, \beta + i; \gamma + i; x]} = 1 - \frac{(\gamma-\beta)(\alpha+i+1)x/(\gamma+i)(\gamma+i+1)}{\frac{2F_1[\alpha+i+1,\beta+i+1;\gamma+i+1;x]}{2F_1[\alpha+i+2,\beta+i+1;\gamma+i+2;x]}}$$

Using equations, we obtain

$$\frac{2F_1[\alpha + i, \beta + i+1; \gamma + i; x]}{2F_1[\alpha + i + 1, \beta + i; \gamma + i; x]}$$

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$$= 1 - \cfrac{(\beta - a)x/(\gamma + i)}{1 - \cfrac{(\gamma-\beta)(a+i+1)x/(\gamma+i)(\gamma+i+1)}{\cfrac{2F_1[\alpha + i + 1, \beta + i+1; \gamma + i+1; x]}{2F_1[\alpha + i + 2, \beta + i+1; \gamma + i+2; x]}}}$$

Again,

$$2F_1[\alpha + i + 2, \beta + i+1; \gamma + i+2; x] - 2F_1[\alpha + i + 1, \beta + i+1; \gamma + i+1; x]$$

$$= \frac{(\gamma-a)(\beta+i+1)x}{(\gamma+i+1)(\gamma+i+2)} \, 2F_1[\alpha + i + 2, \beta + i+2; \gamma + i+3; x]$$

which results,

$$\frac{2F_1[\alpha + i + 1, \beta + i+1; \gamma + i+1; x]}{2F_1[\alpha + i + 2, \beta + i+1; \gamma + i+2; x]}$$

$$= 1 - \cfrac{(\gamma-\alpha)(\beta+i+1)x/(\gamma+i+1)(\gamma+i+2)}{\cfrac{{}_2F_1[\alpha+i+2, \beta+i+1; \gamma+i+2; x]}{{}_2F_1[\alpha+i+2, \beta+i+2; \gamma+i+3; x]}}.$$

We obtain;

$$\frac{2F_1[\alpha + i, \beta + i+1; \gamma + i; x]}{2F_1[\alpha + i + 1, \beta + i; \gamma + i; x]}$$

$$= 1 - \cfrac{(\beta-\alpha)x/(\gamma+i)}{1 - \cfrac{(\gamma-\beta)(\alpha+i+1)x/(\gamma+i)(\gamma+i+1)}{1 - \cfrac{(\gamma-\alpha)(\beta+i+1)x/(\gamma+i+1)(\gamma+i+2)}{\cfrac{{}_2F_1[\alpha+i+2, \beta+i+1; \gamma+i+2; x]}{{}_2F_1[\alpha+i+2, \beta+i+2; \gamma+i+3; x]}}}}$$

Continuing in this manner, we discover the equation.

- **Derivation of the Formulae**

we have

$$\left(\frac{tan^{-1}z}{z}\right)^2 = \frac{1}{\left[1 + \cfrac{z^2}{3+\cfrac{4z^2}{5+\cfrac{9z^2}{7+\cfrac{16z^2}{9+\ldots}}}}\right]^2} = \frac{\left[z - \cfrac{z^3}{3+\cfrac{9z^2}{5+\cfrac{4z^2}{7+\cfrac{25z^2}{9+\cfrac{16z^2}{11+\ldots}}}}}\right]^2}{z^2} = \frac{1}{\left[1 + z^2 + - \cfrac{2z^2}{3-\cfrac{2z^2}{5(1+z^2)-\cfrac{12z^2}{7-\cfrac{12z^2}{9(1+z^2)+\ldots}}}}\right]^2}$$

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$$= \frac{1}{\left[1+\frac{z^2}{1+z^2-2(-1+z^2)+\frac{9z^2}{1+z^2-4(-1+z^2)+\frac{25z^2}{1+z^2-6(-1+z^2)+\frac{49z^2}{1+\ldots+z^2-8(-1+z^2)}}}}\right]^2}$$

again

$$\left(\frac{tan^{-1}z}{z}\right)^2 = F^{1:1;2}_{1:0;1}\left(\begin{matrix}[1:1;1]:[1:1]:\left[\frac{1}{2},1\right],[1:1] & -z^2,-z^2\\ [2:1,1]:-;\left[\frac{3}{2},1\right] & ;\end{matrix}\right)$$

from the above equations we get

$$\frac{1}{\left[1+\frac{z^2}{3+\frac{4z^2}{5+\frac{9z^2}{7+\frac{16z^2}{9+\ldots}}}}\right]^2} = \frac{\left[z-\frac{z^3}{3+\frac{9z^2}{5+\frac{4z^2}{7+\frac{25z^2}{9+\frac{16z^2}{11+\ldots}}}}}\right]^2}{z^2} = \frac{1}{\left[1+z^2+-\frac{2z^2}{3-\frac{2z^2}{5(1+z^2)-\frac{12z^2}{7-\frac{12z^2}{9(1+z^2)+\ldots}}}}\right]^2}$$

$$= \frac{1}{\left[1+\frac{z^2}{1+z^2-2(-1+z^2)+\frac{9z^2}{1+z^2-4(-1+z^2)+\frac{25z^2}{1+z^2-6(-1+z^2)+\frac{49z^2}{1+\ldots+z^2-8(-1+z^2)}}}}\right]^2} =$$

$$F^{1:1;2}_{1:0;1}\left(\begin{matrix}[1:1;1]:[1:1]:\left[\frac{1}{2},1\right],[1:1] & -z^2,-z^2\\ [2:1,1]:-;\left[\frac{3}{2},1\right] & ;\end{matrix}\right)$$

Therefore, the conclusion may be supported. The following formulas may be shown using this method.

5. CONCLUSION

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Since Euler and Gauss, continuous fractions have played a crucial role in number theory and classical analysis. Ordinary and fundamental generalized hyper geometric series have proven a crucial tool in the process of deriving continuing fraction representations. A comprehensive evaluation of the pertinent studies demonstrates that there are basically two solutions to this problem. Using a single three-term recurrence relation between successive hyper geometry functions, the first method determines the continuing fraction representation for the quotient of two hyper geometry series. Utilizing a pair of relations, each of which involves three consecutive hyper geometry series, and then applying the proper transformations to obtain the required representations constitutes the second method. Due to the significance of the properties of approximants of continued fractions and the relationship between those attributes and the values of the functions whose continued fractions they represent, these approximants can also be used to determine the limits of those fractions. In the past four or five decades, however, they have been utilized more frequently as alternative representations for analytic functions than as number theory tools. The works of Ramanujan, which are replete with awe-inspiring continuing fraction representations but make no mention to the number theoretic significance or interpretations of these representations, have been credited with reviving this essential branch of analysis. In an effort to extend the well-known result of Ramanujan using the Lambert series and the continuing fraction that was discovered in the "Lost" notebook, Denis was able to deduce a number of additional astounding results.

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