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AN INVESTIGATION THE FUNCTION OF THE FUNDAMENTAL K-ANALOGUE OF THE GAUSS HYPERGEOMETRIC FUNCTIONS

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ABSTRACT

Hypergeometric functions are taken into account in the theory of special functions, in which case the representations of the functions will be hypergeometric series. The special function theory has historically been used extensively in many fields of mathematical physics, economics, statistics, engineering, and many other scientific disciplines. This work is focused on the investigation of the k-analogue of Gauss hypergeometric functions by the Hadamard product, which was inspired by some recent extensions of the k-analogue of gamma, the Pochhammer symbol, and hypergeometric functions. Additionally, this function yields convergence features. A unique function utilised in mathematics is the Gaussian function, commonly referred to as the common hypergeometric function. The hypergeometric series, which also includes numerous other special functions as specific or limiting examples, serves as a representation of this function. It is the linear solution of a second-order ordinary differential equation (ODE). Any linear ODE of the second order with three regular singular points can be transformed using this equation. In this research, we talk about the invention of a few basic hypergeometric functions. Understanding the importance of the collection of hypergeometric functions in various domains is the goal of this work. This paper's emphasis is on providing background knowledge. New, previously unpublished equations that are cohesively weaved into the body of current mathematical literature make up a large amount of the content.

Keywords: *Hypergeometric, Function, Mathematical, Differential, Equations, etc.*

1. INTRODUCTION

More than 200 years have passed since the first hypergeometric functions with one variable were studied. They have been studied by Euler, Gauss, Riemann, and Kummer, and their findings are available. Schwarz and Goursat investigated the specific characteristics of the variables, whereas Barnes and Mellin studied the integral representations of their variables. The renowned Gauss hypergeometric equation is widely employed in mathematical physics because many well-known partial differential equations can be reduced to Gauss' equation by separating the variables. There are three ways to describe hypergeometric functions: as functions represented by series whose coefficients satisfy specific recursion properties; as solutions to a set of differential equations that are, in the right sense, holonomic

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and have mild singularities; or as functions defined by integrals like the Mellin-Barnes integral. Each of these approaches has Benefits and drawbacks. This interaction is well studied and understood for hyper geometric functions with one variable for many years. On the other hand, when there are several variables, it is possible to expand each of these techniques; however the outcomes may vary slightly depending on which one you select.

As a result, there is no universally accepted definition of what a multivariate hypergeometric function is. One such example is the concept that Horn introduced of multivariate hypergeometric series expressed in terms of the coefficients of the series. As a result of the recursions that they satisfy, a system of partial differential equations is generated. It has come to our attention that for more than two variables, this system does not necessarily need to be holonomic; in other words, the space of local solutions may be of infinite dimensionality. On the other hand, expanding this system of PDEs into a holonomic system can be done in a natural fashion. In the case of two variables, only the relation between these two systems can be grasped in sufficient detail. Even in the case of the classical Horn, Appell, Pochhammer, and Lauricella, multivariate hypergeometric functions, it was not until the 1970s and 1980s that an attempt was made by W. Miller Jr. and his collaborators to study the Lie algebra of differential equations satisfied by these functions and their relationship with the differential equations arising in mathematical physics.

2. CONCEPT OF BASIC HYPERGEOMETRIC SERIES

The Gaussian or regular hypergeometric function 2F1(a,b;c;z) is an example of a specific function that is represented by the hypergeometric series in the subject of mathematics. As specialised or limiting instances, this series also includes a sizable number of extra special functions. It is the linear solution of a second-order ordinary differential equation (ODE). Any linear ODE of the second order with three regular singular points can be transformed using this equation.

2.1 History

John Wallis first used the term "hypergeometric series" in his work Arithmetica Infinitorum, which was published in 1655. The studies of Ernst Kummer (1836) and Bernhard Riemann's essential description of the hypergeometric function by terms of the differential equation it satisfies were both conducted in the nineteenth century. Euler examined hypergeometric series, but Gauss provided the first comprehensive and systematic analysis (1813). The studies of Ernst Kummer were among those conducted in the 20th century (1836). Riemann showed that the three regular singularities of the second-order differential equation (in z) for the 2F1 that was studied on the complex plane could be described. The Annals of the Mathematical Society publication published Riemann's findings.

When the solutions are algebraic functions, H. A. Schwarz identified these instances and created a list of them.

- **<u>The hypergeometric series of equations</u>**

The series for the situation where defines the hypergeometric function. |z|<1.

$$ {}_2F_1(a,b;c;z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!} $$

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assuming c is not equal to 0 or one of the following: -1, -2, or Remember that if either "a" or "b" can be represented as a negative integer, the series ends.

The Pochhammer sign is described as follows:

$(x)_n$=1 if n>0

$x(x+1)\ldots(x+n-1)$)if n>0

Any path in the complex plane outside of the branch points of 0 and 1 can be used to carry out the analysis for different complex values of z.

2.2 Special cases

Numerous additional mathematical functions can be expressed in terms of the hypergeometric function, and limiting examples of it can also be used to do so. Typical examples include the following.

$$ln(1+z) = z\,{}_2F_1(1,1,;2;-z)$$

$$(1-z)^{-a} = {}_2F_1(a,b;b;z)$$

$$arcsin\, z = z\,{}_2F_1\left(\tfrac{1}{2};\tfrac{1}{2};\tfrac{3}{2};z^2\right)$$

The confluent hypergeometric function, commonly referred to as Kummer's function, can be written as a limit of the hypergeometric function.

$$M(a,c,z) = lim_{b\to\infty}\,{}_2F_1(a,b;c;b^{-1}z)$$

Its bounds can therefore be stated as any functions that are essentially special examples of it. Bessel is one such instance of such a function.

functions. This article contains the overwhelming majority of the mathematical and physical functions that are frequently required.

His idea of partitions, which he invented and made famous, has in a natural manner led to series containing components of the form. Euler is credited with developing and popularizing this theory.

$$(l-aq)(l-aq^2)\ldots(l-aq^n)$$

Heine [E. Heine; 1878] conducted the first systematic study of these so-called "basic hypergeometric series" or "Eulerian series." Numerous early findings are attributed to Euler, Gauss, and Jacobi. Bailey [W.N. Bailey, 1935], who has contributed significantly in his own right, offers a succinct summary. It is vital to recognize the contributions of Hahn and Sears to the later, more methodical development of the theory. for both expositions and references that are incredibly thorough. A novice in the field could find the topic of fundamental hyper geometric series to be a little scary due to its extensive development, plenty of potent and universal conclusions, and concise expression. But given the astounding nature of some of the discoveries and their unexpected proximity to the earth's surface, it wouldn't be hard for

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someone to be motivated to carry out their own nearly unexplored investigation.

It seemed inevitable that, when we tackled the subject in this way, we would end up unearthing a lot of knowledge that even the early workers in the field had. However, it was encouraging to observe that many of the discoveries made in this way seemed novel and valuable, whereas earlier discoveries were left behind as convenient by-products.

The analysis of a power series in t with coefficients that each has a single Eulerian factor in the numerator and the denominator has been the focus of our research at least up to this point. This specific action,

$$F\{a,b;\ t) = 1 + \frac{(1-aq)}{(1-bq)}t + \frac{(1-aq)(1-aq^2)}{(1-bq)(1-bq^2)}t^2 + \cdots$$

is an exceptional instance of the Heine series It is capable of satisfying first-order linear difference equations in each of the three arguments, such as

$$(1 - t)F\{a,b;\ t) = (1 - b) + \{b - atq)F(a,b;\ tq)$$

3. BASIC HYPERGEOMETRIC SERIES

As a result of the relatively straightforward q-series that have been taken into consideration up until this point, it has not been necessary for us to develop a condensed notation for q-series that involve multiple parameters. Remember that the technical definition of the hypergeometric series is:

$$F(a,b;c;z) \equiv {}_2F_1(a,b;c;z) \equiv {}_2F_1\left[\begin{matrix}a,b\\c\end{matrix};z\right] = \sum_{n=0}^{\infty}\frac{(a)n(b)n}{n!(c)_n}z^n$$

In this case, it is assumed that c 0, -1, -2... so that none of the terms in the series' denominators include zero factors. The series will then be able to be written as follows: When |z| 1 for the Gauss series, and for |z|=1 when Re(c - a - b) > 0, absolute convergence takes place. Heine was responsible for starting the series.

$$\phi(\alpha,\beta,\gamma,q,z) = {}_2\phi_1(q^{\alpha}, q^{\beta};\ q^{\gamma};\ q,z)$$

With

$${}_2\phi_1(a;b;c;q,z) \equiv {}_2\phi_1\left[\begin{matrix}a,b\\c\end{matrix};q,z\right]\sum_{n=0}^{\infty}\frac{(a;q)n(b;q)n}{(q;q)_n(c;q)_n}z^n$$

where it is taken for granted that $\gamma = -m$ and $c \neq q^{-m}$, where m = 0, 1…. Heine's series converges absolutely for |z|<1 when |q|<1, and it is a q-analogue of Gauss' series because, by applying and setting a formal termwise limit,

$$\lim_{q\to 1^-} {}_2\phi_1(q^{\alpha}, q^{\beta};\ q^{\gamma};\ q,z) = {}_2F_1(\alpha,\beta;\ \gamma;\ z)$$

Heine's series is often referred to as the fundamental hypergeometric series or qhypergeometric series in light of the base q. We prefer to use the $2\phi 1(a,b,c,q,z)$ notation instead of Heine's $\phi(a,b,c,q,z)$ notation because when $0 < |q| < 1$, the limit instances of Heine's series correspond to setting $\alpha, \beta, or\ \gamma \to \infty$ to zero in the corresponding places in the z-axis.

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The (generalised) hypergeometric series with the parameters r numerator $a_1 \dots a_r$ and s denominator $b_1 \dots b_s$ is (officially) defined by

$$_rF_s(a_1, a_2, \dots a_r; b_1 \dots b_s; z) \equiv {}_rF_s \begin{bmatrix} a_1 a_2 \dots a_r \\ b_1 \dots b_s \end{bmatrix}; z = \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n \dots (a_r)_n}{n!(b_1)_n \dots (b_s)_n} z^n$$

and an $_r\phi_s$basic hypergeometric series are described by

$$_r\phi_s(a_1, a_2, \dots a_r; b_1 \dots b_s; z) \equiv {}_r\phi_s \begin{bmatrix} a_1 a_2 \dots a_r \\ b_1 \dots b_s \end{bmatrix}; q, z = \sum_{n=0}^{\infty} \frac{(a_1, a_2 \dots a_r; q)_n}{(q, b_1, \dots, b_s; q)_n} z^n \left[(-1)^n q^{\binom{n}{2}}\right]^{1+s-r} z^n$$

Where $\binom{n}{2} n(n-1)/2$we used the concise notation $(a_1, a_2, \dots, a_r; q)_n = (a_1; q)_n (a_2; q)_n \cdots (a_r; q)_n$**[Gasper; 1990]**

4. HYPERGEOMETRIC FUNCTION

A hypergeometric function is the sum of a hypergeometric series, which is defined as follows.

Definition 1: A series Σc_n is called hypergeometric if the ratio $\frac{c_n+1}{c_n}$ is a rational function of n. This indicates that the factor z arises because the polynomials involved do not have to be monic. The factor (n + 1) in the denominator is useful in the sequel. This factor may or may not be the result of the factorization. If not, one of the components (n + a_i) in the numerator can compensate for this extra factor (select a_i = 1 for some i)

Iteration results in

$$c_n = \frac{(a_1)_n (a_2)_n \dots (a_p)_n z^n}{(b_1)_n (b_2)_n \dots (b_q)_n n!} c_0 n = 1,2 \dots$$

Remember how the shifted factorial $(a)_n$ is well-defined by

$$(a)_n := a(a+1)(a+2)\cdots(a+n-1), n = 1, 2, 3, \dots \text{ and } (a)_0 := 1$$

Hence,

$$\sum_{n=0}^{\infty} c_n = c_0 \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n \dots (a_p)_n}{(b_1)_n (b_2)_n \dots (b_q)_n} \frac{z^n}{n!}$$

Definition 2:A hypergeometric series can be used to define the hypergeometric function pFq(a1, a2,..., ap; b1, b2,..., bq; z), which is written as

$$_pF_q \left(\begin{matrix} a_1 a_2 \dots a_p \\ b_1 b_2 \dots b_q \end{matrix}; z \right)$$

$$= \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n \dots (a_p)_n}{(b_1)_n (b_2)_n \dots (b_q)_n} \frac{z^n}{n!}$$

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It goes without saying that the parameters have to be set up in such a way that the denominator factors in the terms of the series are never zero. If one of the numerator parameters, a_i, is equal to a nonnegative number$-N$, then the hypergeometric function is a polynomial in z. N is an integer that cannot be negative. If that is not the case, then the hyper geometric series has a radius of convergence that is given by

$$p = \begin{cases} \infty & if \quad p < q+1 \\ 1 & if \quad p = q+1 \\ 0 & if \quad p > q+1 \end{cases}$$

This directly follows from the ratio test. Indeed, we have

$$\lim_{n\to\infty}\left(\frac{c_n+1}{c_n}\right) = \begin{cases} 0 & if \quad p < q+1 \\ |z| & if \quad p = q+1 \\ \infty & if \quad p > q+1 \end{cases}$$

The circumstance that $|z| = 1$ is of particular significance when$p = q + 1$. If Re (Pbi Paj) > 0, the hypergeometric sequence ${}_{q+1}F_q(a_1, a_2, \ldots, a_{q+1};\ b_1, b_2, \ldots, b_q;\ z)$ with $|z| = 1$ converges perfectly.

If $|z| = 1$ with $z \neq 1$ and$-1 <$Re $(\Sigma b_i - \Sigma a_j) \leq 0$, the series conditionally converges, and if Re $(\Sigma b_i - \Sigma a_j) \leq -1$, the series diverges.

A generalized hyper geometric function is sometimes used to describe the most universal hyper geometric function, pFq. The term "hyper geometric function" then denotes the unique case.

$${}_2F_1(a, b; c; z) = {}_2F_1\left(\begin{matrix} a & b \\ & c \end{matrix}; z\right) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!}$$

4.1 Generalized hypergeometric function

A generalized hypergeometric function ${}_pF_q(a_1 \ldots . . a_p; b_1 \ldots . b_q; x)$is a function that can be expressed as a hypergeometric series, that is, a series for which the ratio of succeeding terms can be expressed.

$$\frac{c_{k+1}}{c_k} = \frac{P(k)}{Q(k)} = \frac{(k+a_1)(k+a_2)\ldots(k+a_p)}{(k+b_1)(k+b_2)\ldots(k+b_q)(k+1)}\, x.$$

(The presence of the k+1 component in the denominator is due to historical reasons for notation.)

The function ${}_2F_1$ $(a; b; c; x)$ *corresponding to* $p = 2, q = 1$is the first hypergeometric function to be examined (and, in general, emerges the most frequently in physical issues), and as a result, is frequently referred to as "the" hypergeometric equation or, more specifically, Gauss's hypergeometric function. To make matters even more confusing, the phrase "hypergeometric function" is used less commonly to indicate "closed form," and the term "hypergeometric series" is occasionally used to denote "hypergeometric function." Both of these usages are examples of how the terms can be interchanged.

The hypergeometric functions are solutions to the hypergeometric differential equation, which has a regular singular point at the origin. The

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origin is also the location of the regular singular point in the equation. Using the hypergeometric differential equation as a starting point, develop the hypergeometric function.

$$z(1-z)y'' + [c-(a+b+1)z]y' - aby = 0$$

apply the Frobenius technique to condense it to

$$\sum_{n=0}^{\infty}\{(n+1)(n+c)A_{n+1} - [n^2+(a+b)n+ab]A_n\}z^n = 0$$

giving the corresponding equation

$$A_{n+1} = \frac{(n+a)(n+b)}{(n+1)(n+c)}A_n$$

Associating this with the Ansatz series

$$y = \sum_{n=0}^{\infty} A_n z^n$$

the answer is then provided

$$y = A_0[1+\frac{ab}{1!c}z + \frac{a(a+1)b(b+1)}{2!c(c+1)}z^2 + \cdots]$$

This is the so-called regular answer, indicated by

$$ {}_2F_1(a,b;c;z) = 1 + \frac{ab}{1!c}z + \frac{a(a+1)b(b+1)}{2!c(c+1)}z^2 + \ldots$$
$$= \sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(c)_n}\frac{z^n}{n!},$$

This converges if the given value is not a negative integer for all on the unit circle for the given value. A symbol for a Pochhammer can be found here.

The following expression provides a conclusive answer to the hypergeometric differential equation:

The hypergeometric series converges for all arbitrary values, as well as real values and arbitrary values $z = \pm 1$ if $c > a + b$

Derivatives of ${}_2F_1(a,b;c;z)$are assumedby **[Magnus and Oberhettinger 1949]**

$$\frac{d_2F_1(a,b;c;z)}{dz} = \frac{ab}{c}{}_2F_1(a+1,b+1;c+1;z)$$

$$= \frac{ab}{c}{}_2F_1(a+1,b+1;c+1;z)$$

$$\frac{d^2 2F1(a,b;c;z)}{dz^2}$$

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$$=\frac{a(a+1)b(b+1)}{c(c|+1)}{}_2F_1(a+2,b+2;c+2;z)$$

Functions of hypergeometry with specific arguments can be reduced to functions of elementary geometry, for instance.

$${}_2F_1(1,1;1;z)=\frac{1}{1-z}$$

$${}_2F_1(1,1;2;z)=\frac{1n(1-z)}{z}$$

$${}_2F_1(1,2;1;z)=\frac{1}{(1-z)^2}$$

$${}_2F_1(1,2;2;z)=\frac{1}{1-z}$$

5. HYPERGEOMETRIC SERIES AND DIFFERENTIAL EQUATION

Equations: The Gamma Function, in addition to the Pochhammer Symbol. We are reminded that the integral can be used to define the Gamma function, which is denoted by the symbol (s).

$$\Gamma(s)=\int_0^{\infty}e^{-t}t^{8-1}dt$$

A holomorphic function in the half-plane Re(s) > 0 is defined by the integral. In addition, it answers the functional equation.

$$\Gamma(s+1)=s\Gamma(s);\ \ Re(s)>0$$

Hence, since $\Gamma(1)=1$, we have $\Gamma(n+1)=n!$ for all $\in\mathbb{N}$. We can expand to a meromorphic function in the entire complex plane with simple poles at non-positive integers. For instance, in the strip $\{-1<Re(s)\leq 0\}$, we define

$$\Gamma(s)=\frac{\Gamma(s+1)}{s}$$

Definition 3: Given α $\in\mathbb{C}\backslash\mathbb{Z}_{\leq 0}$ and k $\in\mathbb{N}$The Pochhammer symbol is defined:

$$(a)_k=\frac{\Gamma(a+k)}{\Gamma(a)}$$

5.1 Hypergeometric Series

Let $n=(n_1,\ldots,n_r)\in\mathbb{N}^r$ be an r-tuple of non-negative integers.

Given $x=(x_1,\ldots,x_r)\in\mathbb{C}r$,

We shall indicate the power product by x^n.

$$x^n := x^{n1}_1\cdot\cdot\cdot x^{nr}_r,$$

and by ej the j-th standard basis vector in Qr .

Definition 4: A formal multivariate power series

$$F(x_1,\ldots,x_r)=\sum_{n\in\mathbb{N}^r}A_nx^n$$

is only considered to be (Horn) hyper geometric if the quotient is valid for all j = 1,..., r.

$$R_j(n):=\frac{A_{n+e_j}}{A_n}$$

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A rational function of $n = (n_1 . . n_r)$

Example: Assume we want $R(n) = R_1(n)$ to be a constant function and set r = 1. Then, for some, $A^n = A_0^{cn}$; c ∈ℂ and therefore

$$F(x) = A_0 \sum_{n=0}^{\infty} c^n x^n = \frac{A_0}{1 - cs}$$

As a result, in its most basic form, a hypergeometric series is just a geometric series.

5.2 Differential Equations

The fact that the coefficients of a hypergeometric series recur implies that these coefficients are formal solutions to either ordinary or partial differential equations. In the first step of this process, we will derive the ordinary differential equation of the second order that is satisfied by Gauss' hypergeometric function. The following notation will be utilised for the rest of this discussion: We will use x for the differentiation operator d/dx when dealing with functions that have just one variable$x$. When dealing with functions that have several variables, such as $x_1, \dots, x_n$, we will write j for the partial differentiation operator $\partial/\partial x_j$. In addition to this, we will look at the Euler operators:

$$\theta_x := x\partial_x \, ; \, \theta_j : = x_j \partial_j$$

Now consider the Gauss hyper geometric series, where F is substituted for $_2F_1$ to simplify the notation. We possess

$$\theta_x F(\alpha, \beta, \gamma; x) = \sum_{n=0}^{\infty} \frac{(\alpha)_n (\beta)_n}{(\gamma)_n n!} \, n \, x^n$$

However, according to the exercise:$k\ (\alpha)k = \alpha\ ((\alpha + 1)k - (\alpha)k)$

$n(\alpha)_n = \alpha((\alpha + 1)_n - (\alpha)_n)$ and therefore

$$\theta_x F(\alpha, \beta, \gamma; x) = \sum_{n=0}^{\infty} \left(\frac{(\alpha + 1)_n (\beta)_n}{(\gamma)_n n!} - \frac{(\alpha)_n (\beta)_n}{(\gamma)_n n!} \right)$$

$$= a(Fa + 1, \beta, \gamma; x) - F(a, \beta, \gamma : x))$$

[E. L. Ince, 1944] Henceforth;$(\theta x + \alpha) \cdot F(\alpha, \beta, \gamma;\ x) = \alpha \cdot F(\alpha + 1, \beta, \gamma;\ x)$

6. GENERALIZATIONS OF THE HYPERGEOMETRIC FUNCTION

Among the generalisations of the hypergeometric function are the following:

- A generalisation of hypergeometric series based on two variables, the Appell series
- Basic hypergeometric series in which the ratio of components is a periodically changing function of the index
- Series of bilateral hypergeometric functions PHp are analogous to generalised hypergeometric series, except their sums are performed on all integers.
- A type of elliptic hypergeometric series in which the ratio of terms is expressed as an elliptic function of the index

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- In addition to the Meijer G-function, the Fox H-function also plays a role.
- Function of Fox and Wright, which is a generalization of the generalized hypergeometric function
- Generalized hypergeometric series denoted by the notation pFq, in which the ratio of terms is a rational function of the index
- Function of the Heun, solutions of the second order ODEs that have four constant single points.
- 34 different converging hypergeometric series in two variables make up The Horn Function.
- The hypergeometric function of a matrix argument is a generalization of the hypergeometric series that may be applied to several variables.
- The Lauricella hypergeometric series is a hypergeometric series that consists of three variables.
- The MacRobert E-function is an extension of the generalized hypergeometric series p_{Fq} to the case in which p is greater than or equal to q+1.
- Meijer G-function is an extension of the generalized hypergeometric series p_{Fq} to the case where p is greater than or equal to q+1.

7. CONCLUSION

Hypergeometric functions could appear while studying fractional calculus. The way these functions are carried out depends on the data (such fractionally integrated data) from time series and other branches of economics. A final observation on hypergeometric functions. They are currently included in a wide range of computer programmes due to their increasing importance in a wide range of mathematical applications. These software programmes include Mathematica and Maple, both of which let users manipulate symbols. One of its most important benefits, in addition to the general simplicity with which they approach problems, is their ability to offer clear solutions.

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Author's Declaration

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