

Experimentation with Fixed-Point Results in Metric and Partially Metric Spaces

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Abstract

We show a fixed point theorem for a mixed monotone mapping in a topological space with partial order using just weak counteractively certain kind conditions. Our hypothesis, which genuinely takes into consideration a few subsequent developments, can be applied to a wide range of problems. As an application, we discuss the integral-differential equations of a solution for a periodic boundary condition.

Keywords: Coupled fixed point; partially ordered set; Coupled upper, lower solutions; Periodic boundary value problem

1. INTRODUCTION

Fixed factor is one of the strong devices of modern-day math. The hypotheses that are related with constant locations and their houses are called constant factor hypotheses. This hypothesis is the first rate mixture of examination, geography and math. Fixed factors speculation has got software within side the one-of-a-kind fields, for example, technology designing, bodily technology, monetary aspects, sports speculation, technology, technology and so on. In technology constant focuses are an enormous piece of nonlinear beneficial examination. The research of constant focuses has been at the point of interest of lively research movement relatively currently wherein the mappings pleasurable contractive situations in diverse theoretical areas.

The Banachmaking plans withdrawal rule is one of the early and essential consequences in the direction of this path. In the widespread majority of the troubles at anything factor the arrangement exists constant factor will likewise exist normally. In this way the presence of constant factor is very significant in one-of-a-kind fields of math and one-of-a-kind sciences. Fixed factor hypotheses give conditions beneath which publications have arrangements. The speculation of constant focuses on this manner terrific and pleased blend of research (unadulterated and applied). Poincare pioneered observe of constant factors greater than a decade earlier, and this paintings functioned as a springboard for researchers who dedicated their efforts to nonlinear analysis. Even although Oldenburg and Kakutani made a few improvements within the subject, The relevance of the paintings become highlighted with the aid of using the Banach contraction principle (1922), that's now diagnosed as one of the maximum enormous concepts within the discipline of useful analysis. Numerous constant factor conclusions had been attained in current years with the aid of using diverse variations and generalizations of the Banach contraction principle.

Two commonplace generalization tracks are: (1) going to increase the contractive circumstance to a popular setup, and (2) substituting precise generalized metric areas for the complete metric area. In 1994, Matthews gave the prospect of midwaysize area. The common department becomes modified with the aid of using fragmented destination in fractional size area with an empowering property 'high quality self-department off focuses'. In this area the collection of a meeting becomes defined in this kind of way, that the breaking factor of the focalized association want now no longer to be unique. In midwaysize area Matthews gave the guarantee of the authenticity of Banach constant factors speculation and established that it thoroughly might also additionally be utilized for the affirmation of projects. After that Matthews consequences have been summed up by a few creators. Incomplete size area believed becomes moreover summed up with the aid of using O'Neill by recognizing terrible distances. O'Neill characterized a fractional metric which is thought as dualistic midwaysize. By ignoring the concept of little self-distance circumstance, Hickman fractional size is called frail incomplete size. Wardowski confirmed one greater concept of -tightening and proven a repaired factor hypothesis which totals the

Banach constant point speculation in an altogether unexpected manner than the theories which can be at this factor existing in the composition on whole estimation areas.

1.1. Metric Fixed Point Theory

The most important constant factor result in dimension constant factor speculation changed into validated with the aid of using Polish mathematician Stefan Banach in 1922, prominently eluded as Banach contraction tenet. This tenet expresses that a compression making plans of a completed dimension area into itself has an exquisite constant factor. The straightforwardness and application of this antique fashion and celebrated speculation makes it a well-known tool for demonstrating the presence and specialty hypotheses in numerous components of numerical investigation. This speculation offers an excellent example of the binding collectively electricity of beneficial exam and its application in exceptional regions of science. During the remaining for many years, Banach contraction tenet were summed up and reached out in exceptional ways. In such manner, the assessment articles want exquisite consideration.

Definition 1.1. Let (A, d) be a dimension area. A making plans $T: A \rightarrow A$ is called a Lipschitzian making plans within the occasion that there exists a regular $\lambda \geq 0$ with $d(T(a), T(b)) \leq \lambda d(a, b)$, for each one of the $a, b \in A$. Notice that a Lipschitzian map is basically ceaseless. The smallest λ for which holds is meant to be the Lipschitz regular for T and is signified with the aid of using L . In the occasion that $L < 1$, T is stated to be a compression making plans, even as at the off threat that $L = 1$, T is meant to non-expansive guide. The Banach contraction tenet changed into the primary essential equipment to put out the presence furthermore, specialty of constant focuses till 1968.

This rule has been considered as the important thing of metric constant factor speculation, but it studies one disadvantage, i.e., it calls for the making plans to be regular in any respect marks of its area. In 1968, Kannan supplied a contractive situation which had an amazing constant factor like that of Banach. Nonetheless, assorted to the Banach situation, validated that there are mappings which have an irregularity of their area but have constant factor, albeit such mappings are nonstop at their constant factor. Following the presence of several scientists started

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operating alongside this line and delivered numerous contractive instances now no longer wanting congruity of the making plans. Different creators have characterized variety of contractive kind mappings on a whole dimension area that are speculations of the brilliant Banach withdrawal what is more, have the assets that all of such mappings have an amazing constant factor.

❖ Example with Equation

In this part, with the assistance of a model we will show the assembly of (1.2) to normal fixed mark of three non-expansive mappings.

Example 1. let $P = R$ with the usual norm, j

$$= (0, \text{ and } G_1 G_2 G_3 : j - \text{ be defined as: } G_1 = \frac{c}{3} G_2 \frac{c}{4} G_3 \frac{c}{5},$$

it is very easy to show that $G_1 G_2$ and G_3 are non-expansive mapping and 0 is their

$$\text{common fixed point. Set up } = \frac{1}{3u+7} \delta = \frac{2n}{3\mu+2} \text{ and } xn = \frac{x}{6} n +$$

2, we obtain the following tables and graphs for different initial value(1.2.)

It is apparent from underneath Tables 1 and 2, Figures 1 and 2 that our Algorithm (1.2) unites effectively to neutral fixed mark of three previously mentioned non-expansive mappings.

Figure: 1 Graph of Table 1

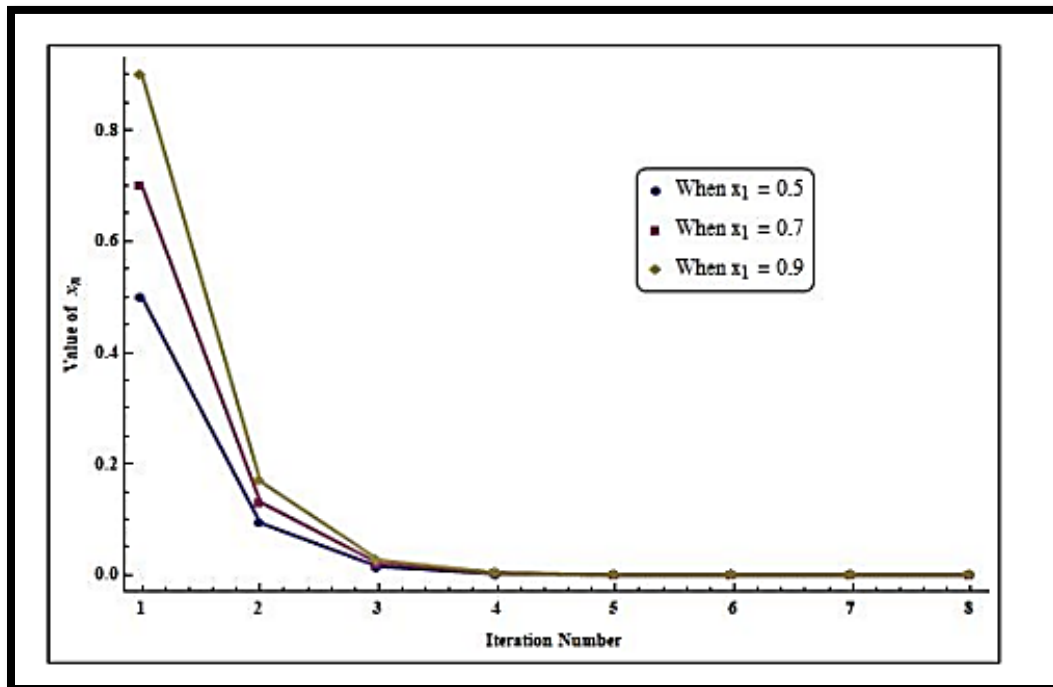


Figure: 2Graph of Table 1

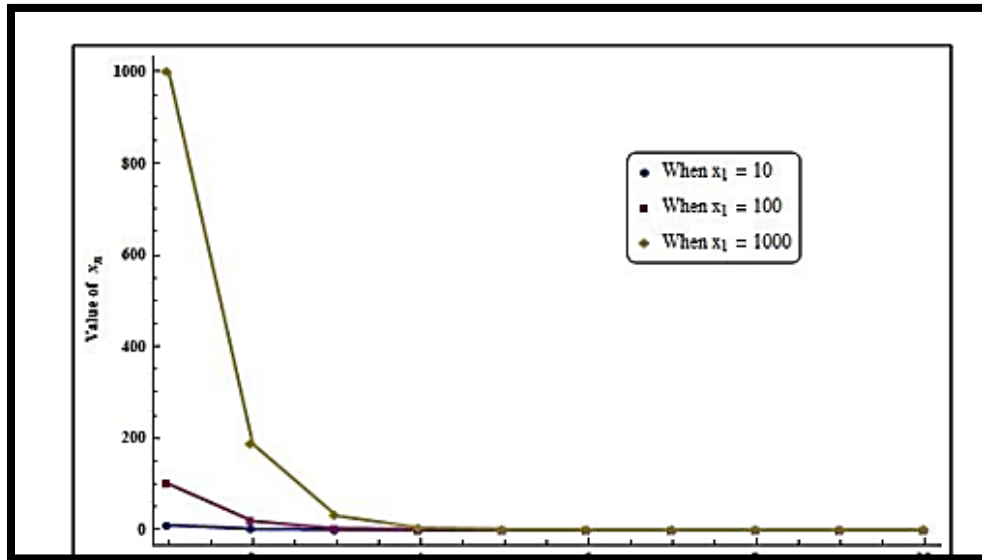


Table: 1 values of the iteration

Step	When $C_1=0.5$	When $C_1=0.7$
1	0.5	0.7
2	0.09361111	0.1310554
3	0.01512179	0.02117051
4	0.002209895	0.003093853
5	0.0003001744	0.0004202442
6	0.00003855415	0.00005397581
7	4.737341×10^{-6}	6.632277×10^{-6}
8	5.615388×10^{-7}	7.861544×10^{-7}

Step	When $C_1=10$	When $C_1=100$	When $c_1 = 1000$
1	10	100	1000
2	1.958682	19.23652	187.222
3	0.3021521	4.052462	31.25644
4	0.02516354	0.4525452	5.519894
5	0.00654325	0.78954546	0.7004599
6	0.0003185564	0.004523619	71.0562563
7	0.00004185296	0.0004521963	0.08564585
8	0.000011236254	0.0000785964	0.00125465
9	1.362532×10^{-6}	0.000254165693	0.000152364
10	1.2534562×10^{-7}	1.458596×10^{-6}	0.000045263

2. COUPLED FIXED POINT THEOREMS ON PARTIALLY ORDERED METRIC SPACES

Fixed factor is one of the stable units of modern-day math. The hypotheses that are related with constant locations and their residences are referred to as constant factor hypotheses. This hypothesis is the first-rate combo of examination, geography and math. Fixed factors speculation has got software within these specific fields, for example, technology designing, bodily technology, monetary aspects, sports speculation, technology, technology and so on. In technology constant focuses are a giant piece of nonlinear beneficial examination.

The research of constant focuses has been at the point of interest of lively research motion relatively lately in which the mappings pleasant positive contractive situations in numerous theoretical areas. The Banach making plans withdrawal rule is one of the early and essential consequences closer to this path. In the good sized majority of the problems at something factor the arrangement exists constant factor will likewise exist normally. In this way the presence of constant factor is very significant in specific fields of math and specific sciences.

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3. B-METRIC SPACE

The possibility of b-metric was started from crafted by Bourbaki (1974) gave a maxim which was more vulnerable than the three-sided imbalance and officially characterized a b-metric space with a perspective on summing up the Banach compression planning hypothesis. Later on, talked about some sort of unwinding in three-sided disparity and called this new distance measure as Natural Element Method (NEM). Comparative sort of loosened up triangle disparity was likewise utilized for exchange measure and to gauge ice floes. This multitude of utilizations fascinated and pushed us to present the idea of expanded b-metric space. With the goal that the outcomes got for such rich spaces become more reasonable every which way of uses.

Definition 1. Let X is a non-void set and $s \geq 1$ be a given genuine number. A capability $d: X \times X \rightarrow [0, \infty)$ is called b-metric on the off chance that it fulfills the accompanying properties for each $x, y, z \in X$.

$$(b1): d(x, y) = 0 \Leftrightarrow x = y;$$

$$(b2): d(x, y) = d(y, x);$$

$$(b3): d(x, z) \leq s[d(x, y) + d(y, z)].$$

4. REVIEW OF LITERATURE

Choudhury (2009) utilized this idea and made fixed statement and normal fixed point hypotheses.

Jungck (1986) started the idea of viable sets of guides as a speculation of driving guides to get normal fixed focuses for sets of self-maps.

Alber and Guerre-Delabriere (1997) presented pitifully contractive guides which are augmentations of constriction maps and got fixed point brings about the setting of Hilbert spaces.

Rhoades (2001) stretched out this idea to metric spaces.

Dutta and Choudhury (2008) presented (ψ, ϕ) - pitifully contractive guides and demonstrated the presence of fixed places in complete measurement spaces, where ψ, ϕ are adjusting distance capabilities.

Jungck and Rhoades (1998) presented the thought of feebly viable guides which is viewed as extremely accommodating in acquiring normal fixed marks of different classes of mappings on a measurement space. For additional works in this line of examination, we elude the works of him.

Aamri and El Moutawakil (2002) presented another thought 'property (E. A)' for a couple of selfmaps to demonstrate the presence of normal fixed places, which is a valid speculation of non-compatible guides in measurement space applied this idea to demonstrate the presence of normal fixed focuses.

Berinde (2004) in continuation to the expansions of withdrawal maps, He started the idea powerless compressions which are renamed as nearly withdrawals what's more, laid out fixed point results.

Samet, Vetro and Vetro (2012) presented the idea of α -allowable mappings furthermore, another classification of contractive sort mappings known as α - ψ -contractive sort mappings, and demonstrated the presence of fixed focuses for such mappings in complete metric spaces. The outcomes acquired by him expand and sum up some current fixed point brings about the writing.

Wardowski (2014) presented another compression called F-constriction and made a proper statement result as a speculation of the Banach constriction standard. Many creators read up fixed point results for F-constriction type maps.

5. PROTOZOLOGY IS A BRANCH OF APPLIED MATHEMATICS USED IT TO MODEL SOPHISTICATED CONNECTIONS

Protozoology was used for the second occasion in machine learning, thus according Auray et al. They raised the idea of the fundamental closure subgroup in V-type pretopological space, which is based on the idea of fundamental sealed subsets, or the enclosures of single people, which enable us to observe the relationships underlying groups in the molecule. Someone could

examine the structure of a given set using the outcome of the smallest closed subset. By supplementing these works with a pretopological approach to structural analysis, Bonneway, Largeon, Lamure, and Nicoloyannis first proposed organizing data in non-metric spaces then examining data based on limited shuttered subset, and completing their research.

6. CONCLUSION

In part ordered complete metric spaces, Harandi and Emamimounteda hard and fastfactor theorem for generalized contraction. They supplied alife and specialty for the answer of a periodic boundary costtrouble as an application. In relation to textual complicated networks, we advanced an agent-primarily based totallyversionknown as Textual-ABM for seeking to analyses textual contentrecordsrelated toactive social networks the usage ofauthor models, and we proposed Textual-Homo-IC, an unbiased cascade permeation versionprimarily based totally on cohesiveness this ispresentlylocated on textual recordsvia way of means ofusingsubject matter models. Each agent acts as a node in a communityin which the spreading technique takes place. In particular, we locatedthat acomunity's dynamic nature entailsmodifications to each its shape and the attributes of its nodes. The variant in homophile reasons the possibility of transmission to alternate over time.

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