AIRO JOURNALS

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Peer Reviewed
Multidisciplinary
National

ISSN: 2321-3914
Volume:4 Issue: 2
November 2021
Impact Factor:5.7
Subject Mathematics

ON DERIVATION IN NEAR-RINGS AND ITS GENERALIZATIONS: A SURVEY

MOHD SAJID SHAMIM
MATHEMATICS

Abstract

We present an historical account of the study of derivations, generalized derivations, n-derivations, generalized n-derivation and other kinds of derivations in near-rings, based on the work of several authors. Moreover, recent results on semigroup ideals and generalized n-derivations on these topics have been discussed in details. Examples of various notions have also been included.

Keywords: Derivation, Near-Rings, Generalization

Introduction

The present paper is an attempt to present an up-to-date account of work on derivations and its various invariants in the setting of near-rings. The work has been presented in a manner suitable for everybody who have some basic knowledge in near-ring theory. In order to make the treatment as self-contained as possible, and to bring together all the relevant material in a single paper, we have included several references. Some times, many results have been unified in a single theorem. Proper references of almost all the results are given. Let N be non empty set, equipped with two binary operations say '+' and '.'. N is called a left near-ring if (i)(N , +) is a group (not necessarily abelian) (ii)(N , .) is a semigroup and (iii)x.(y + z) = x.y + x.z for all x, y, z ∈ N . Similarly a right near-ring can also be defined. A left near-ring N is called zero-symmetric if 0.x = 0 for all x ∈ N (recall that in a left near ring x.0 = 0 for all x ∈ N). Similar remarks hold for a right near-ring also. For a natural example of a near-ring, let (G, +) be a group (not necessarily abelian). Consider S, the set of all mappings from G to G. Then S is a zero-symmetric right near-ring with regard to the operations '+' and ' . ' defined as below:

where f, g ∈EndG. It is to be noted that it is not a left near-ring.

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ISSN: 2321-3914
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Derivations in Near-Rings

The notion of derivation in rings is quite old and plays a significant role in various branches of mathematics. It has got a tremendous development when in 1957, Posner [39] established two very striking results on derivations in prime rings. Also there has been considerable interest in investigating commutativity of rings, more often that of prime ring and semiprime rings admitting suitable constrained derivations. Derivations in prime rings and semiprime rings have been studied by several algebraists in various directions. Motivated by the concept of derivation in rings Bell and Mason [24] introduced the concept of derivation in near-rings as following. Definition 2.1. A derivation ′d ′ on N is defined to be an additive mapping $d : N \to N$ satisfying the product rule $d(xy) = xd(y) + d(x)y$ for all $x, y \in N$.

Example 2.2.Let $N = N1 \oplus N2$, where N1 is a zero symmetric left near-ring and N2 is a ring having derivation δ. Then $d : N \longrightarrow N$ defined by $d(x, y) = (0, \delta(y))$ for all $x, y \in N$ is a nonzero derivation of N , where N is a zero-symmetric left near- ring.

For an example of a derivation on noncommutative near-ring one can consider the following:

Example 2.3. Let us consider $(C, +, *)$ where ′∗ ′ is defined as $x * y = |x|y$ for all $x, y \in C$, then it can be easily seen that $(C, +, *)$ is a zero-symmetric left near-ring which is not a right near -ring. Assume N =

$$\left\{\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}\right\} | \alpha, b \ \in \}$$

then N is a zero-symmetric left near-ring which is not a right near-ring. Define $d : N \longrightarrow N$ as d

$$\left\{\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}\right\} = \left\{\begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix}\right\}$$

Then d is a non-zero derivation on N . In a left near-ring, right distributive property does not hold in general, the following lemmas play a vital role in further study. For any $a, b, c \in N$ expanding $d(a(bc))$ and $d((ab)c)$ and comparing the relations so obtained we get the following (for reference see ([24], Lemma 1)).

Lemma 2.4. Let d be an arbitrary derivation on a near-ring N . Then N satisfies the following partial distributive law:

ISSN: 2321-3914
Volume:4 Issue:2
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$$(ad(b) + d(a)b) = ad\,(b)c + d(a)bc\,for\ all\ a, b, c \in \mathbf{N}.$$

The study of derivation was initiated by H. E. Bell and G. Mason [24], pertaining to the 3-prime near-rings and semiprime near-rings. Some basic properties of 3-prime near-rings are given below which are helpful in the study of derivations in 3-prime near-rings:

- If $z \in Z \setminus \{0\}$, then z is not a zero divisor.
- If Z contains a nonzero element z for which $z + z \in Z$, then (N, +) is abelian.
- Let d be a nonzero derivation on N. Then $xd(N) = \{0\}$ implies $x = 0$ and $d(N)x = \{0\}$ implies $x = 0$.
- If N is 2-torsion free and d is a derivation on N such that d 2 = 0, then d = 0.

In the year 1984 X.K.Wang ([41], Proposition 1) gave an equivalent definition of derivation on a near-ring N as below and also obtained partial commutativity of addition and partial distributive law in the near-ring N .

Definition 2.5. Let d be an arbitrary additive endomorphism of N . Then d is a derivation on N

if $d(xy) = d(x)y + xd(y)$ for all $x, y \in N$.

Lemma 2.6. Let d be a derivation on N .Then N satisfies the following partial distributive law:

$(d(x)y + xd(y))z = d(x)yz + xd(y)z$ for all $x, y, z \in N$

Lemma 2.7. Let N be a near-ring with center Z, and let d be a derivation on N . Then $d(Z) \subseteq Z$.

Major study in this area was carried out by Bell and Mason [24], Beidar, Fong and Wang [16] etc. which consists of commutativity of addition and multiplication of 3-prime near-ring and semiprime near-ring with constrained derivations. It has been also studied that under suitable constrained derivations, 3-prime near-rings behave like rings.

Now we list several commutativity theorems, obtained by above authors for 3-prime near-rings, admitting suitable constrained derivations as below.

Results given below have been proved by Bell and Mason [24].

Theorem 2.8.

If a 3-prime near-ring N , admits a non trivial derivation satisfying either of the following properties

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ISSN: 2321-3914
Volume:4 Issue:2
November 2021
Impact Factor:5.7
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(i) d(N) ⊆ Z,

(ii) [d(x), d(y)] = 0 for all x, y ∈N , then (N , +) is abelian and if N is 2-torsion free as well, then N is a commutative ring.

Following results concerning commutativity of near-ring have been proved by Beidar, Fong and Wang [16]

Theorem 2.9. Let N be 3-prime near-ring which admits derivations d1 and d2. Suppose N satisfies any one of the following properties:

$$(1) d_1^2 \neq 0 \neq d_2^2 and\ d_1(x)d_2(y)d_2(y)d_2(y)d_1(x) for\ all\ x, y \in n.$$

$$(11)\ 2n \neq 0, d_1 \neq 0, d_2 \neq 0 and\ d_1 0\ and\ d_1(x)d_2(y) = d_2(y)d_1(x) for\ all\ x, y \in n.$$

$$then\ n\ is\ a\ comminutative\ ring.$$

Theorem 2.10. Let N be 3-prime near-ring with nonzero derivations d1 and d2 such that d1(x) d2(y) = −d2(x) d1(y) for all x, y ∈N . Then (N , +) is abelian.

Very recently Boua and Oukhtite [25] investigated some differential identities which force a 3-prime near-ring to be a commutative ring and also gave the suitable examples, proving the necessity of the 3-primeness condition.

Theorem 2.11. ([25], Theorem 2.2-2.3). Let N be a 3-prime near-ring. Suppose that N admits a nonzero derivation d satisfying the following property, i.e.; d ([x, y]) = ±[x, y] for all x, y ∈N. Then N is a commutative ring.

Remark 2.12. The following example shows that the 3-primeness in the hypothesis of the above theorem is essential even in the case of arbitrary rings.

Examples 2.13let R be a commutative ring which is not a zero ring and consider $\left\{\begin{pmatrix} 0 & 0 \\ x & y \end{pmatrix}\right\} \mid 0, x, y, \in R$, if we define d: n ⟶ n by d $\begin{pmatrix} 0 & 0 \\ x & y \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ x & 0 \end{pmatrix}$, then it is straightforward to check that d is a nonzero derivation of n. on the other hand , if a = $\begin{pmatrix} 0 & 0 \\ x & y \end{pmatrix}$

, where 0 ≠ r, then a Na = {0} which proved that N is not 3- prime. Moreover, d satisfied the condition d [A, B] = [A, B] for all A, B €n is not a commutative ring.

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ISSN: 2321-3914
Volume:4 Issue:2
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Theorem 2.14. ([22], Theorem 2.2-2.3). Let N be a 2-torsion free 3-prime near-ring. If N admits a nonzero derivation d satisfying any one of the following properties:

$$[\boldsymbol{d}(\boldsymbol{x}), \boldsymbol{y}] = [\boldsymbol{x}, \boldsymbol{d}(\boldsymbol{y})] \text{ } \boldsymbol{for\ all\ x, y} \in \boldsymbol{N},$$

$$[\boldsymbol{d}(\boldsymbol{x}), \boldsymbol{y}] = \pm [\boldsymbol{x}, \boldsymbol{d}(\boldsymbol{y})] \text{ } \boldsymbol{for\ all\ x, y} \in \boldsymbol{N},$$

$$[\boldsymbol{x}, \boldsymbol{d}(\boldsymbol{y})], = [\boldsymbol{d}(\boldsymbol{y})\ )] \text{ } \boldsymbol{for\ all\ x, y} \in \boldsymbol{N},$$

$$[\boldsymbol{x}, \boldsymbol{d}(\boldsymbol{y})], = -[\boldsymbol{d}(\boldsymbol{y})\ )] \text{ } \boldsymbol{for\ all\ x, y} \in \boldsymbol{N},$$

$$\boldsymbol{Then\ n\ is\ a\ commentative\ ring}.$$

Generalized Derivations in Near-Rings

Matej Bresar [27] introduced the concept of generalized derivation in associative rings. This concept covers the concept of derivation already known to us for ring theory. Later a lot of study was done by Hvala, Golbasi, T. K. Lee etc. about generalized derivations in the setting of prime rings and semiprime rings and several known results for derivation in prime and semi prime rings were extended in the setting of generalized derivations in rings by above authors.

Motivated by the above concept, Golbasi[28] introduced the concept of generalized derivations in near-rings as given below and studied this in the setting of 3-prime and semi prime near-rings. Later in 2008, H. E. Bell[19] also studied this notion and derived some commutativity theorems of 3-prime near-rings equipped with generalized derivation. The above authors also generalized the several known results of derivations in 3-prime and semiprime near-rings.

Definition 3.1. Let N be a near-ring. An additive mapping f : N −→ N is called

(i) a right generalized derivation of N if there exists a derivation d of N such that f(xy) = f(x)y + xd(y) for all x, y ∈N .

(ii) (ii) a left generalized derivation of N if there exists a derivation d of N such that f(xy) = d(x)y + xf(y) for all x, y ∈N .

(iii) (iii) a generalized derivation of N if there exists a derivation d of N such that f(xy) = f(x)y+ xd(y) for all x, y ∈ N and f(xy) = d(x)y + xf(y) hold for all x, y ∈N .

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ISSN: 2321-3914
Volume:4 Issue:2
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On n- derivations in near-rings

Recently K. H. Park [36] introduced the notion of an n-derivation and symmetric n-derivation, where n is any positive integer in rings and extended several known results, earlier in the setting of derivations in prime rings and semiprime rings. Motivated by the above notion in rings the authors [5] introduced the notion of n-derivations in the setting of near-rings and generalized several known results obtained earlier in the setting of 3-prime near-rings and semiprime nearrings.

Definition 4.1

$\boldsymbol{A\ map\ d}: \frac{n\times n\times \ldots . x\ n \to n}{n-times}$ ***is said to be permuting if the equation*** $\boldsymbol{d(x1, x2, \ldots\ldots\ldots xn) = d(xn(1)\ x - n \ldots\ldots\ldots . x1 \ldots x, n.}$ ***is called where sn is the permutation group on*** $\{1, 2, \ldots\ldots, n\}$. ***a map*** $\boldsymbol{d: n \to n}$ ***defined by*** Ω €

Sn, where sn, s the permutation group on $\{1,2, \ldots\ldots, n\}$.a map d: permutation map, is called the trance of d.

Definition 4.2let n be any fixed positive interior. An n- additive (I,e) : additive in eachf$(x)y + xd(y)$ $\boldsymbol{for\ all\ x, y\ \in\ N\ argument)\ mapping\ D: N \times N\ x\ \ldots . x\ N \to n\ is\ called\ an\ n-\ derivation\ on\ n\ if\ recreations\ D(X1\ X2\ \ldots\ldots\ X1. <}$ $X.1\ iX\ \ldots\ldots . . <, = D(x, X2,) = \boldsymbol{X,)\ N, i = 1, 2, 3, \ldots\ldots n \ldots ., \ if\ in\ addition, (x'1, x2, \cdot \cdot \cdot, xn\ D\ is\ a}$

permutation map then the above conditions are educement and in this case D is called a permutations n- permutation n- derivation of new permutations n- derivations of n- additive permutation mapping below : ..

$\boldsymbol{An\ n-\ if\ D}\ (\boldsymbol{x}, \ldots . \boldsymbol{1\ x} \ldots \boldsymbol{2} < \cdots .)\ xi\ \ldots .) = D\ (x, 1\ x, 2)\ \boldsymbol{D(X, \ldots\ldots\ldots X2)'\ x, \ldots\ldots\ldots\ldots\ )\ of\ n\ if\ holds\ for\ all\ x, 1\ X, 2}$, € Ň.

$\boldsymbol{Lemma\ 4.7. Let\ N\ be\ a\ nearring. Then\ D\ is\ a\ permuting\ n-derivation\ of\ N\ if\ and\ only\ if\ D(x1x_d^{'}1, x2, \cdot \cdot \cdot, xnx1D(x'1, x2, \cdot \cdot \cdot, xn) + D(x1, x2, \cdot \cdot, xn)x'1\ for\ all\ x1, x1', x2, \cdot \cdot \cdot, xn\ \in\ N.}$

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In a left near-ring N , right distributive law does not hold in general, however, the following partial distributive properties in N have been obtained in ([5], Lemma 2.4-2.6).

Theorem 4.8. Let N be a near-ring. Let D be a permuting n-derivation of N and d be the trace of D. Then

(i) $\{D\,(x1\,,x2,\ldots.Xn\,)\,X,;\,'\;+\;\{D\,(x1\,,x2,\ldots.Xn\,)\,X,;\,')\,\}\}\,y\;=$

(ii) $\{D\,(x1\,,x2,\ldots.Xn\,)\,X,;\,'\;+\;\{D\,(x1\,,x2,\ldots.Xn\,)\,X,;\,')\,\}\}\,y\;=$

$for\ every\ x1, x1\,.', \ldots\ xny \in N.$

(iii) $\{\,X,\ldots\ D\,,(\,x1\,,x2,\ldots.Xn\,)\,X,;\,'\;+$

$\{D\,(\,x1\,,x2,\ldots.Xn\,)\,X,;\,')\,\}\}\,y\;=$

(iv) $\{\,X,1\ \ldots\ D\,,(\,x1\,,x2,\ldots.Xn\,)\,X,;\,'\;+$

$\{D\,(\,x1\,,x2,\ldots.Xn\,)\,X,;\,')\,\}\}\,y\;=\;for\ every\ x,1\ x,2\ \ldots\ x,n\ y \in\ N.$

(v) $\{D\,(x)\,x1\,,\ldots.1\ x\ldots2\ X2\ \ldots.)\,xi\ \ldots.)\;=\;D\,(\,x,1\ x,2)\;+$

$D(\,X,\ldots\ldots\ldots X2\,)'\,x,\ldots\ldots\ldots\ldots\ )$

(vi) $\{\,X1\ \ D\,(x)\,x1\,,\ldots.1\ x\ldots2\ X2\ \ldots.)\,xi\ \ldots.)\;=\;D\,(\,x,1\ x,2)\;+$

$D(\,X,\ldots\ldots\ldots X2\,)'\,x,\ldots\ldots\ldots\ldots for\ every\ x,..X,..\in\ N.$

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ISSN: 2321-3914
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(vii) $\boldsymbol{If\ N\ is\ 3-\ prime\ , D\ \neq}$

$\boldsymbol{0, and\ xD\ (n, N, \ldots\ldots\ldots\ldots.N\ )\ x\ \{0\}\ where\ x \in\ N.\ then\ x, = 0}$.

(viii) $\boldsymbol{If\ N\ is\ 3-\ prime\ , D\ \neq}$

$\boldsymbol{0, and\ xD\ (n, n, \ldots\ldots\ldots\ldots.N\ )\ x\ \{0\}\ where\ x \in\ N.\ then\ x, = 0}$.

(ix) $\boldsymbol{(9) If\ N\ is\ 3-\ prime\ , D\ \neq\ 0, and\ xC\ (n, C\ , \ldots C\ \ \ldots\ldots\ldots.C\ \ ) \neq}$

$\boldsymbol{x\ \{0\}\ where\ \ \ C\ \ \ where\ C \neq\ \{0\}}$.

Recently Öztürk and Jun ([35], Lemma 3.1) proved that in a 2-torsion free 3-prime near-ring which admits a symmetric bi-additive mapping D if the trace d of D is zero, then D = 0. Further, this result was generalized by K.H. Park and Y.S. Jun ([37], Lemma 2.2) for permuting tri-additive mapping in 3!-torsion free 3-prime near-ring. We have extended this result, as below, for permuting n-additive mapping in a n!-torsion free 3-prime near-ring under some constraints.

On Generalized n-Derivations in Near-rings

Motivated by the concept of generalized derivation in rings and near-rings the authors [10] generalized the concept of n-derivation of near-rings by introducing the notion of generalized derivations in near-rings.

Definition 5.1.

Let n be a fixed positive integer. An n-additive mapping $F : N \times N \times \cdots \times N \longrightarrow N$ is called a right generalized n-derivation of N with associated n-derivation D if the relations

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$$F(x1, x2, \ldots., xi - , x, x, i+ , \ldots \ldots\ xn\ ) = F\ (x1, x2, \ldots., xi - , x, x, i+ , \ldots \ldots\ xn\ )x, n + +x, 1\ x, D\ (X1, x2, \ldots..xi\ x, i..+1\ , \ldots..,X\ n,)$$

hold for all

$$(x1, x2, \ldots., xi - , x, i\ \ x, i\ , x, i + , \ldots \ldots\ xn\ ) = \ xn, € , i = 1, 2, 3, \ldots., n,$$

If in addition, both F and D are permuting maps then all the above conditions are equivalent and in this case F is called a permuting right generalized n-derivation of N with associated permuting n-derivation D. An n-additive mapping F: N × N × · · · × N −→ N is called a left generalized n-derivation of N with associated n-derivation D if the relations

$$F(x1, x2, \ldots., xi - , x, x, i+ , \ldots \ldots\ xn\ ) = F\ (x1, x2, \ldots., xi - , x, x, i+ , \ldots \ldots\ xn\ )x, n + +x, 1\ x, D\ (X1, x2, \ldots..xi\ x, i..+1\ , \ldots..,X\ n,)$$

Hold for all

$$(x1, x2, \ldots., xi - , x, i\ \ x, i\ , x, i + , \ldots \ldots\ xn\ ) = \ xn, € , i = 1, 2, 3, \ldots., n,$$

If in addition, both F and D are permuting maps then all the above conditions are equivalent and in this case F is called a permuting left generalized n-derivation of N with associated permuting n-derivation D. An n-additive mapping $F : N \times N \times \cdot\cdot\cdot \times N \longrightarrow N$ is called a generalized n-derivation of N with associated n-derivation D if it is both a right generalized n-derivation as well as a left generalized n-derivation of N with associated n-derivation D. If in addition, both F and D are permuting maps then F is called a permuting generalized n-derivation of N with associated permuting n-derivation D (see [10] for further reference). If N is a commutative ring, then it is trivial to see that the set of all left generalized n-derivations of N is equal to the set of all right generalized n-derivations of N .

Semigroup ideals and generalized n-derivations in near-rings

A nonempty subset A of N is called semigroup left ideal (resp. semigroup right ideal) if N A ⊆ A (resp. AN ⊆ A) and if A is both a semigroup left ideal and a semigroup right ideal, it will be called a semigroup ideal. Recently many authors have studied commutativity of addition and ring behavior of 3-prime near-rings satisfying certain properties and identities involving derivations and generalized derivations on semigroup ideals (see [2],[18],[32][33], where further references can be found). In the present section we study

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Volume:4 Issue: 2
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the commutativity of addition and ring behavior of 3-prime near-rings satisfying certain properties and identities involving generalized n-derivations on semigroup ideals. In fact, the results presented in this section generalize, extend, compliment and improve several results obtained earlier on derivations, generalized derivations, permuting n-derivations and generalized n-derivations for 3-prime near-rings; for example Theorem 1.2 of [2], Theorems 3.2−3.4&3.7 of [5], Theorems 3.1, 3.11, 3.15, 3.16 of [10], Theorems 3.2 − 3.3 of [18] etc.- to mention a few only. We begin with the following theorem obtained in ([12], Theorem 3.1).

Theorem 6.1. Let N be a 3-prime near-ring and A1, A2, ..., An be nonzero semigroup ideals of N . If it admits a nonzero generalized n-derivation F with associated n-derivation D of N such $\boldsymbol{that\ F(A1, A2, \dots, An)}$ ⊆Z, then N is a commutative ring. Corollary 6.2. ([10], Theorem 3.1). Let N be a 3-prime near-ring admitting a nonzero generalized n-derivation F with associated n-derivation D of N . If F(N , N , · · · , N) ⊆ Z, then N is a commutative ring. The following example demonstrates that N to be 3-prime is essential in the hypothesis of the above theorem.

Theorem 6.4. ([12], Theorem 3.2). Let N be a 3-prime near-ring and A1, A2, · · · , An nonzero semigroup ideals of N . If it admits generalized n-derivations F and G with associated nonzero n-derivations D and H of N respectively such that

F $\boldsymbol{(x1, x2, \cdots, xn)H(y1, y2, \cdots, yn) = -G(x1, x2, \cdots, xn)D(y1, y2, \cdots, yn)}$

$$\boldsymbol{for\ all\ x1, y1 \in A1;\ x2, y2 \in A2; \cdots;\ xn, yn \in An, then\ (N, +)\ is\ abelian.}$$

Corollary 6.5. ([10], Theorem 3.15). Let F and G be generalized n-derivations of 3-prime nearring N with associated nonzero n-derivations D and H of N respectively such that

$$\boldsymbol{F(x1, x2, \cdots, xn)H(y1, y2, \cdots, yn) = -G(x1, x2, \cdots, xn)D(y1, y2, \cdots, yn)}$$

$$\boldsymbol{for\ all\ x1, x2, \cdots, xn, y1, y2, \cdots, yn \in N. Then\ (N, +)\ is\ an\ abelian\ group.}$$

Let X and Y be nonempty subsets of N and a ∈N . By the notations [X, Y] and [X, a] we mean the subsets of N defined by [X, Y] = {[x, y] | x ∈ X, y ∈ Y } and [X, a] = {[x, a] | x ∈ X} respectively. Very recently A. Ali et al. ([2], Theorem 12) proved that if N is a 3-prime near-ring, admitting a nonzero generalized derivation f with associated nonzero derivation d

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such that [f(A), f(A)] = {0}, where A is a nonzero semigroup ideal of N , then (N , +) is abelian. We have improved and extended this result for generalized n-derivation in the setting of 3-prime near-rings. In fact we obtained the following. Theorem 6.6. ([12], Theorem 3.3). Let N be a 3-prime near-ring and A1, A2, · · · , An nonzero semigroup ideals of N . If it admits generalized n-derivations F1 and F2 with associated nonzero n-derivations D1 and D2 of N respectively such that [F1(A1, A2, · · · , An), F2(A1, A2, · · · , An)] = {0}, then (N , +) is abelian. Corollary 6.7. ([10], Theorem 3.16). Let F1 and F2 be generalized n-derivations of 3-prime near-ring N with associated nonzero n-derivations D1 and D2 of N respectively such that [F1(N , N , · · · , N), F2(N , N , · · · , N)] = {0}. Then (N , +) is an abelian group. The following example shows that the restriction of 3-primeness imposed on the hypotheses of Theorems 6.2 & 6.3 is not superfluous.

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ISSN: 2321-3914
Volume:4 Issue: 2
November 2021
Impact Factor:5.7
Subject Mathematics

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